DT:- 8/9/2020

Module-1

Structural analysis-II

→ It is the determination of the effects of loads on physical structure and their components.

→ Structure subjected to all this type of analysis include all that must withstand loads, such as buildings, bridges aircraft and ships.

→ The result of the analysis are used to verify a structure's fitness for use.

Introduction

If a structure is determinate then equations of equilibrium are enough to find reactions shear force, bending moment and axial thrust at supports and member.

If the structure is indeterminate then apart from equilibrium condition extra compatibility condition are required which depends upon the properties of material and cross section

Methods for SA-I

Strain energy method

Method of consistent deformation

Three Moment Theorem.

Virtual work / unit load method.

Method of minimum potential energy

Column analogy method.

Methods of SA-II

Moment distribution method

Kani's method

Slope deflection meth

Slope deflection Method of Analysis

This Method was given by G.A Maney.

→ This Method is based on stiffness approach and basic unknowns are taken as joint displacements (θ & Δ).

→ To find unknowns (joint displacements) joint Moment equilibrium conditions and shear equations are written and the joint Moment in members are found by force / displacement relation called as slope deflection equation

Note :- In this Method deformation due to bending are only considered and axial deformation are neglected.

Assumptions :-

(1) All joints are rigid, i.e the angle between any two members in a joint doesn't change even after deformation due to loading.

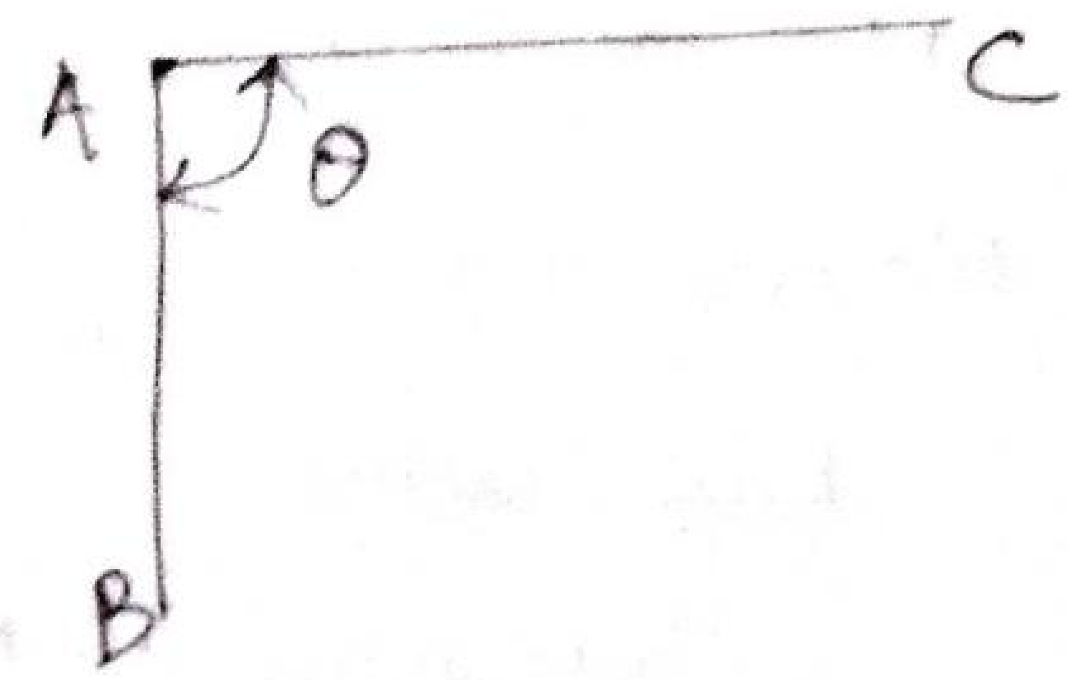
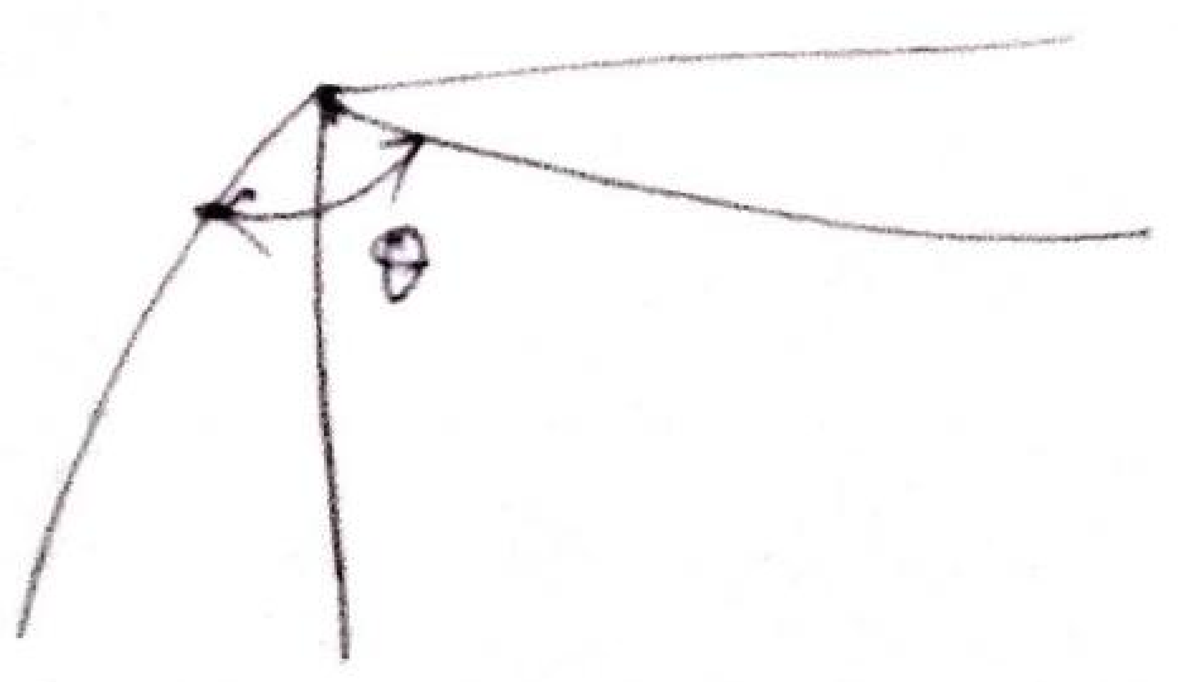


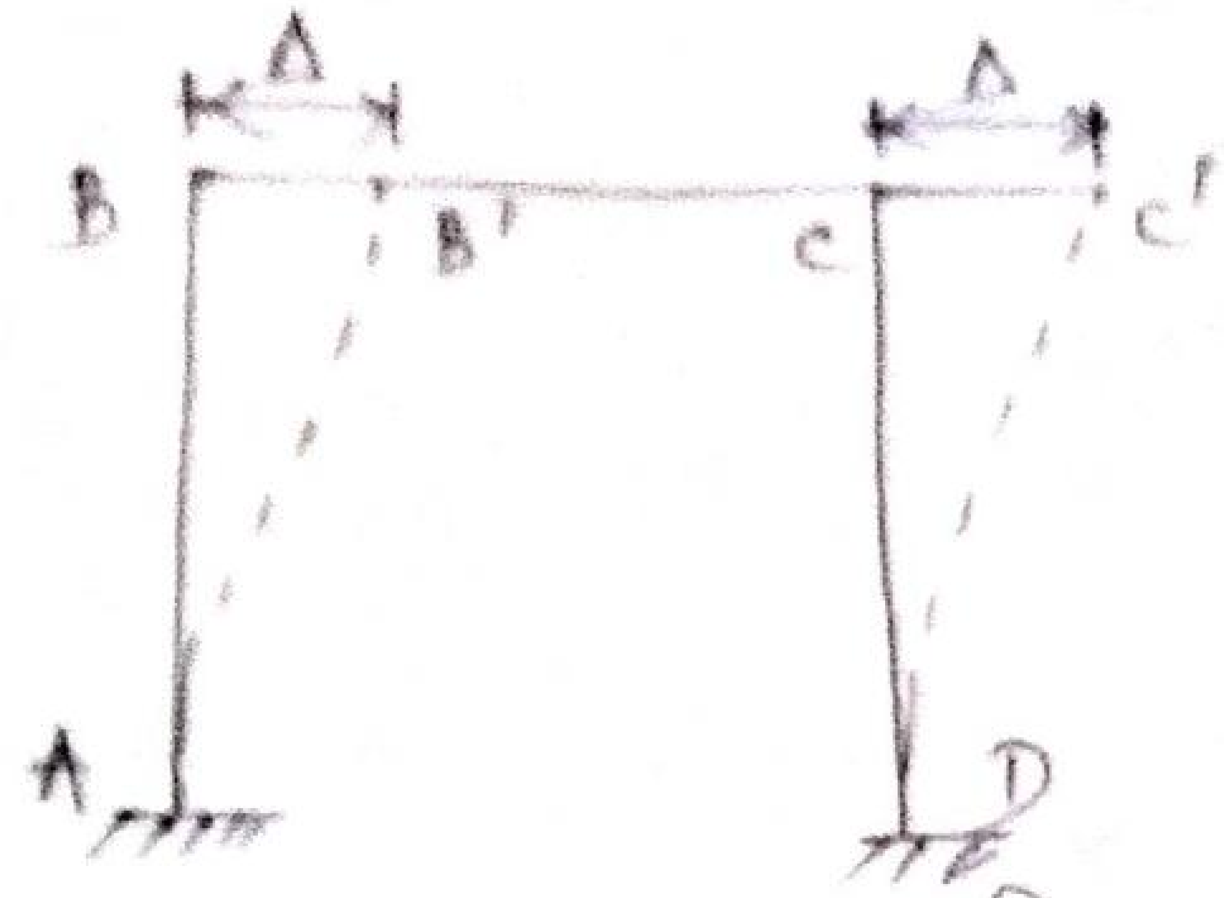
fig (a) :- In this figure the angle between AB and BC remains θ before deformation



This figure shows rigid joint after deformation.

→ The angle between AB and BC remains θ after deformation.

(2) Distortions due to axial and shear stresses are very small. So neglected.



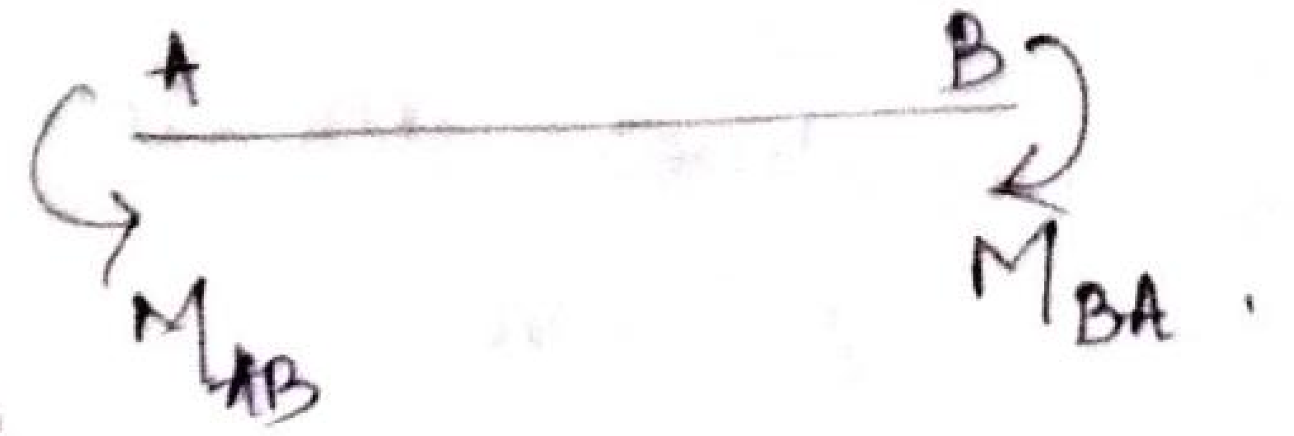
$BB' = CC' = \Delta$

Axial deformation neglected.

Note: Distortion of rigid jointed structure due to axial forces and shears are really small compared to distortion due to flexure.

Signconvention:

(1) Endmoments :-

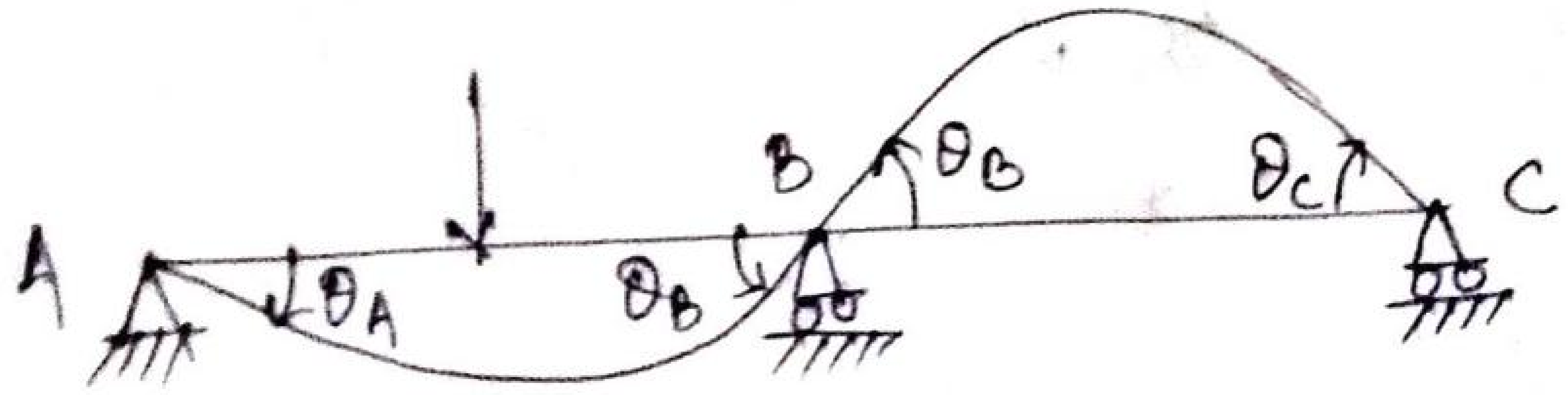


Clockwise Moments are taken as positive and anticlockwise Moments are taken as -ve.

Here $M_{AB} = -ve$ (anticlockwise)

$M_{BA} = +ve$ (clockwise).

(2) Slope (Rotation).



Clockwise rotations are taken as positive and anticlockwise Rotation are taken as -ve.

Here $\theta_A = +ve$ (clockwise)

$\theta_B = -ve$ (anticlockwise)

$\theta_C = +ve$ (clockwise).

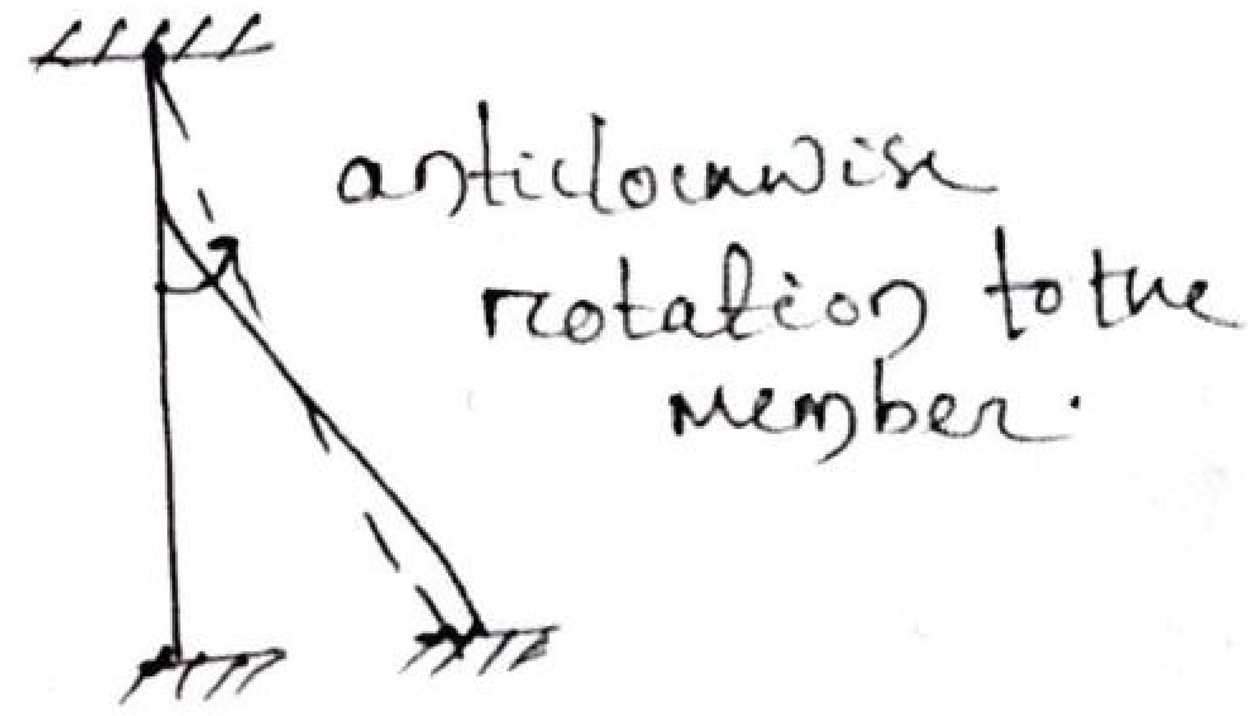
(c) Deflection (settlement) :-

(i) Those displacement (deflections) will be positive which produces clockwise rotation to the member.

Here $\Delta = +ve$.

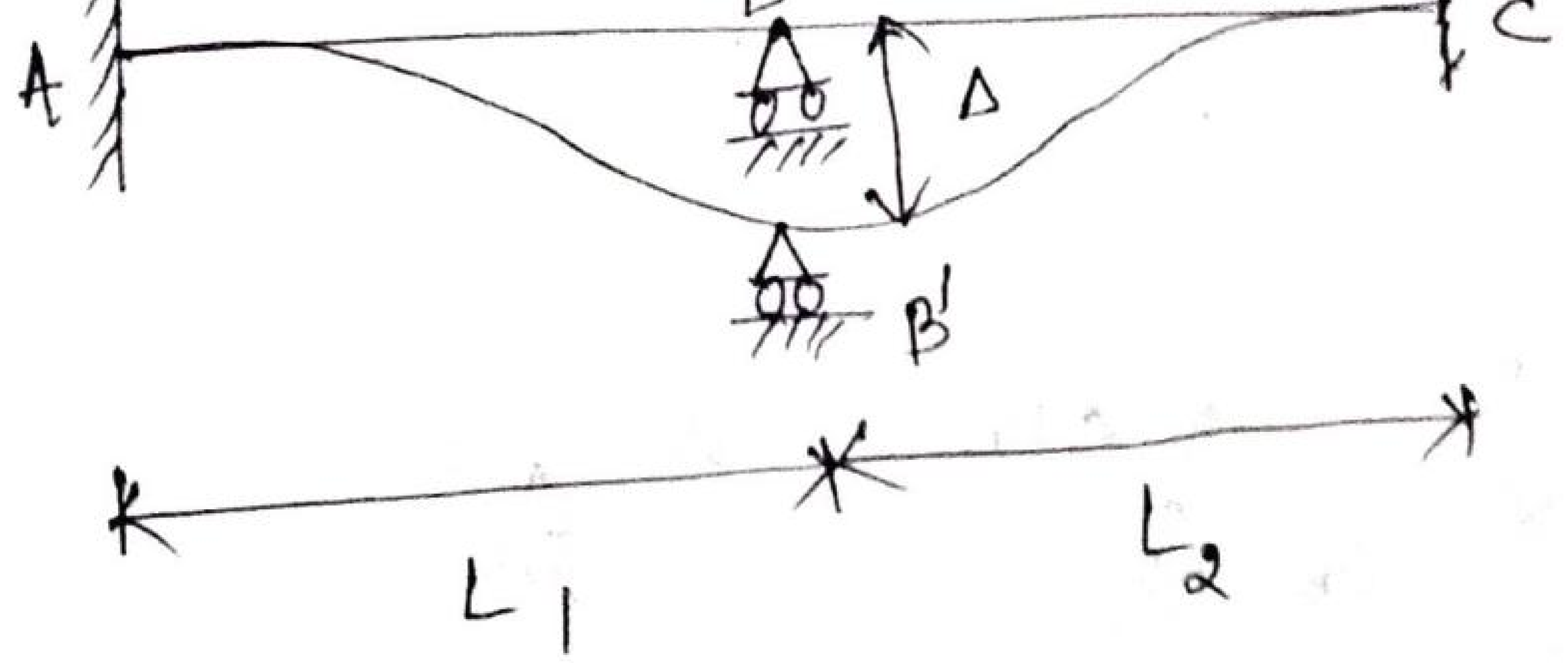
(ii) Negative deflection :- Those displacements will be negative which produces anticlockwise rotation to the member.

Here $\boxed{\Delta = -ve}$.



Other method.

(1.) Settlement Δ is +ve if right side support is below leftside support.



(ii) Δ is considered as (-ve) if left side support is below the right side support.

Thus in this figure for beam AB Δ is positive and for beam BC Δ is negative.

* $\boxed{M_A = 4EI/_L\, \theta_A}$

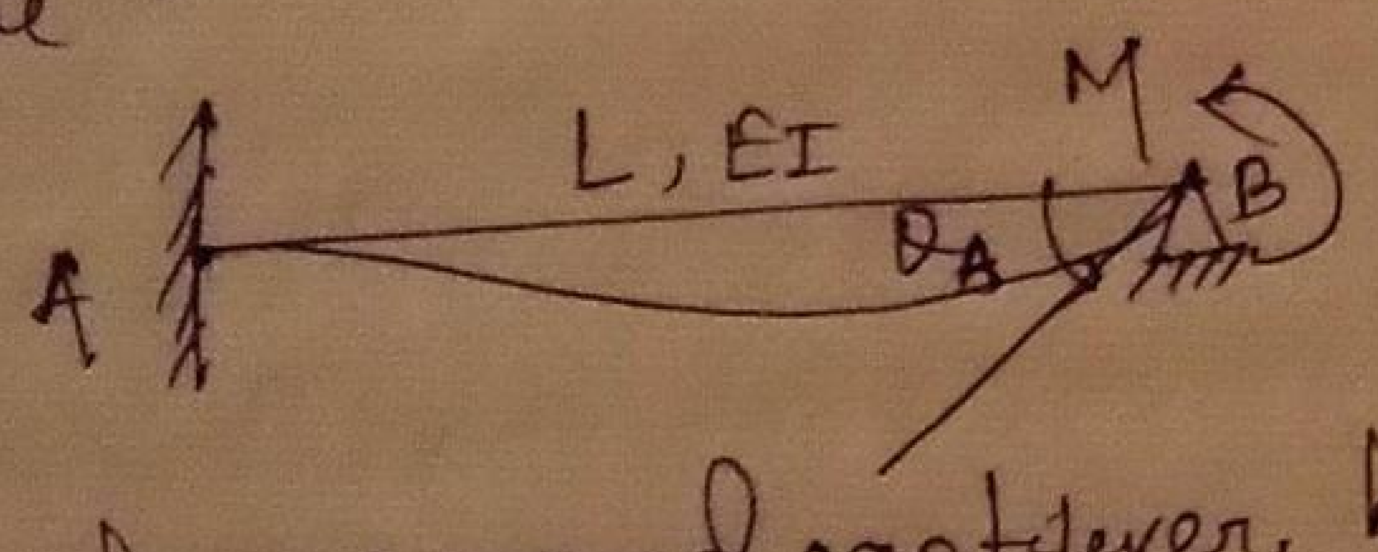
$$\text{Stiffness} = \frac{M_A}{\theta_A} = \frac{\frac{4EI}{L}\cancel{\theta_A}}{\cancel{\theta_A}} = 4EI/_L.$$

Stiffness :-

Stiffness for a member at a joint is the moment (force) required to produce unit rotation (displacement) at that joint.

→ Stiffness at a joint depends upon end condition and properties of crossection.

Consider a propped cantilever beam AB as shown in figure.



for above propped cantilever beam if anticlocwise Moment is applied at end B The rotation at B is given by

$$\boxed{\theta_B = \frac{ML}{4EI}} \Rightarrow M = \frac{4EI}{L}\theta_B$$

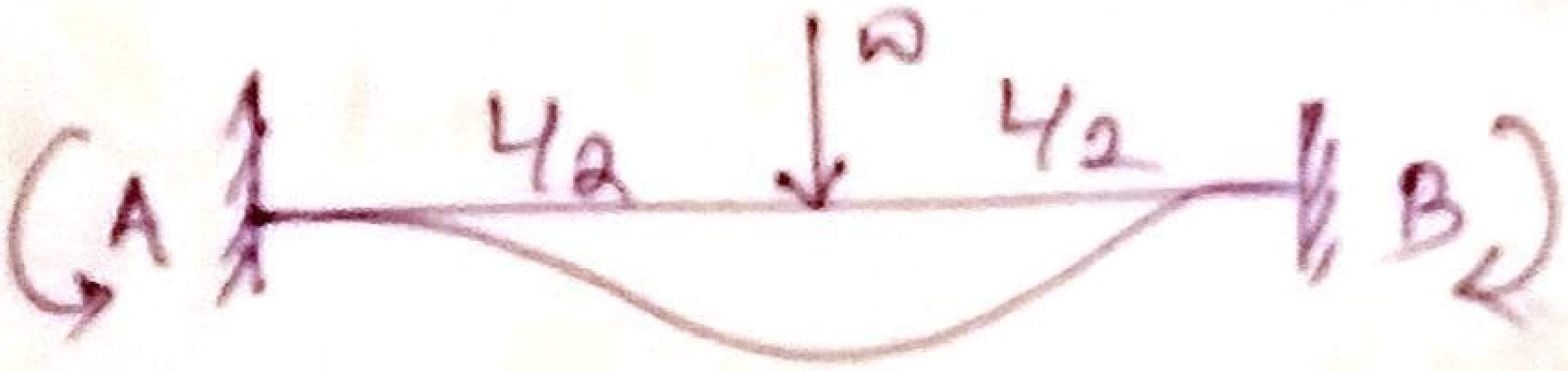
we k/w Moment required to produce unit rotation is stiffness

$$K_{BA} = \frac{M}{\theta_B} = \frac{4EI/_L\, \cancel{\theta_B}}{\cancel{\theta_B}} = 4EI/_L.$$

* Fixed end Moment.

The fixed end Moments are reaction moments developed in a beam member under certain load conditions with both end fixed.

→ If we take a fixed beam which is subjected to a point load at centre of distance (L)



→ At point 'A' an anticlockwise resisting Moment developed and at point 'B' a clockwise resisting moment developed.

→ At 'A' Moment will be M_{FAB} ↺ (-)

'B' " " M_{FBA} ↻ (+)

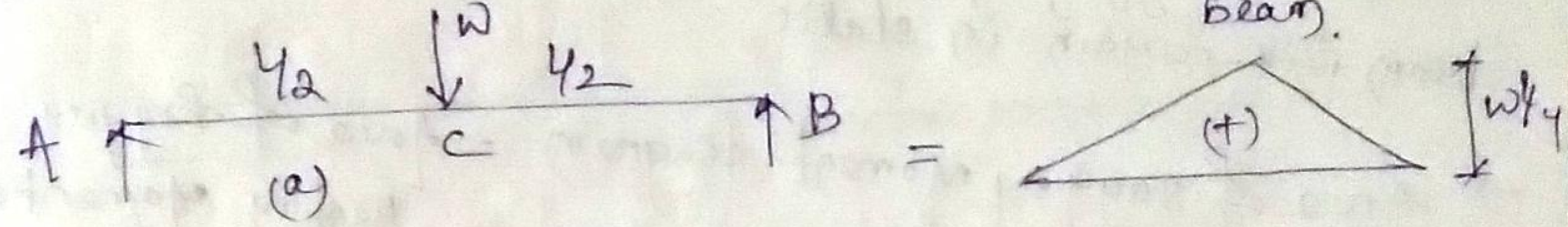
for fixed beam carrying a concentric point load at centre

$$M_{FAB} = -wL/8$$

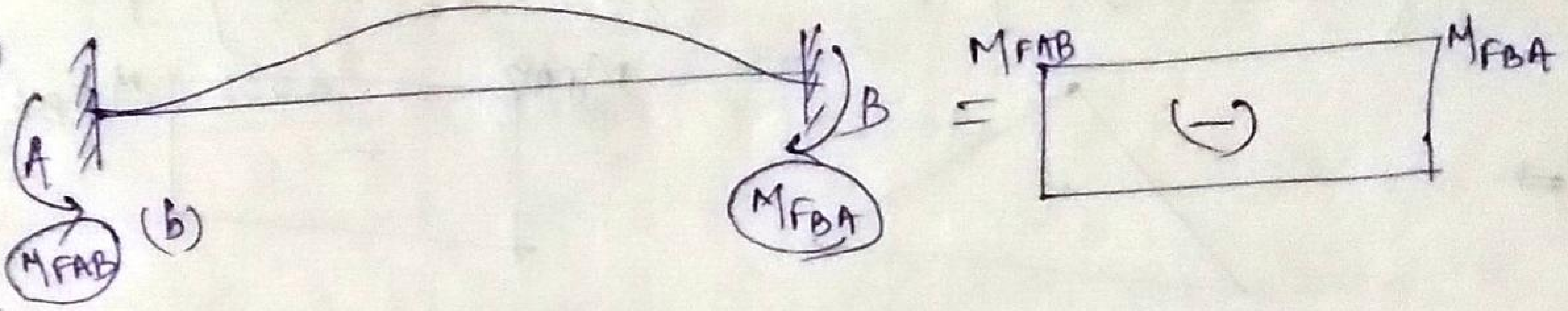
$$M_{FBA} = +wL/8$$

As loading is concentric and symmetric both the value are same but if the loading is unsymmetric then the value for both supports are different whereas sign will be different.

Explanation :- Convert the fixed beam into simply supported beam.



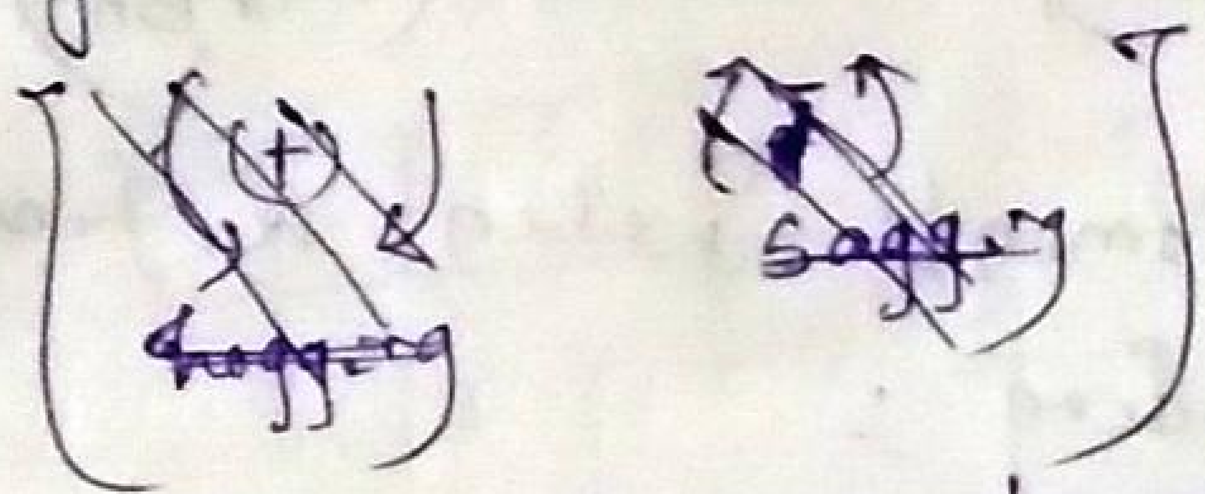
Again



Note :-

In simply supported beam as the beam is sagging so sagging moment (WL/4) will develop. (Refer fig-a)

→ In figure (b) the nature of fixed end moment is hogging.



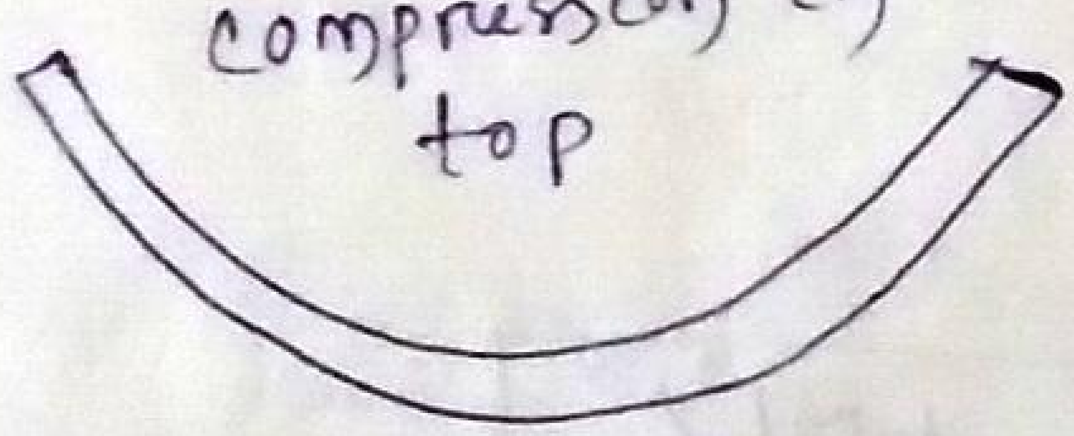
That means the end moments are taking the beam upward.

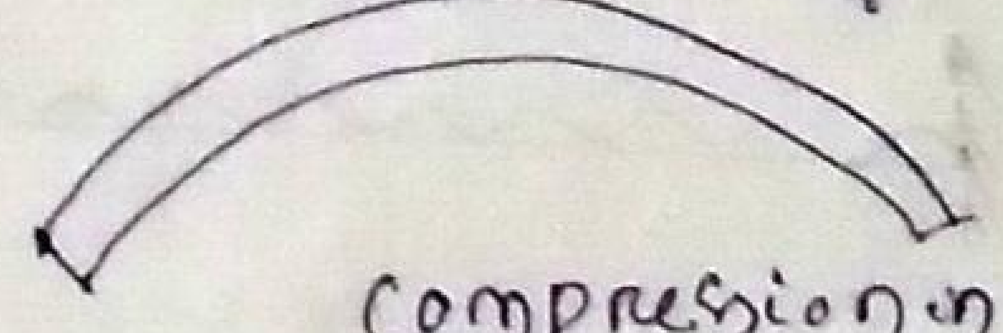
$M_{FAB} = M_{FBA}$ so the bending moment diagram will be rectangular.

→ If the beam wouldn't symmetric then $M_{FAB} \neq M_{FBA}$.

Tension in top

compression in top





compression in bottom

Tension in bottom

(Hogging)

[Sagging]

* In reality the amount of sagging and hogging of beam is very very less that's why the beam will remain in static.

If the sagging moment = hogging moment, then the beam will remain in static.

→ Area of sagging moment diagram = Area of hogging Bending moment diagram

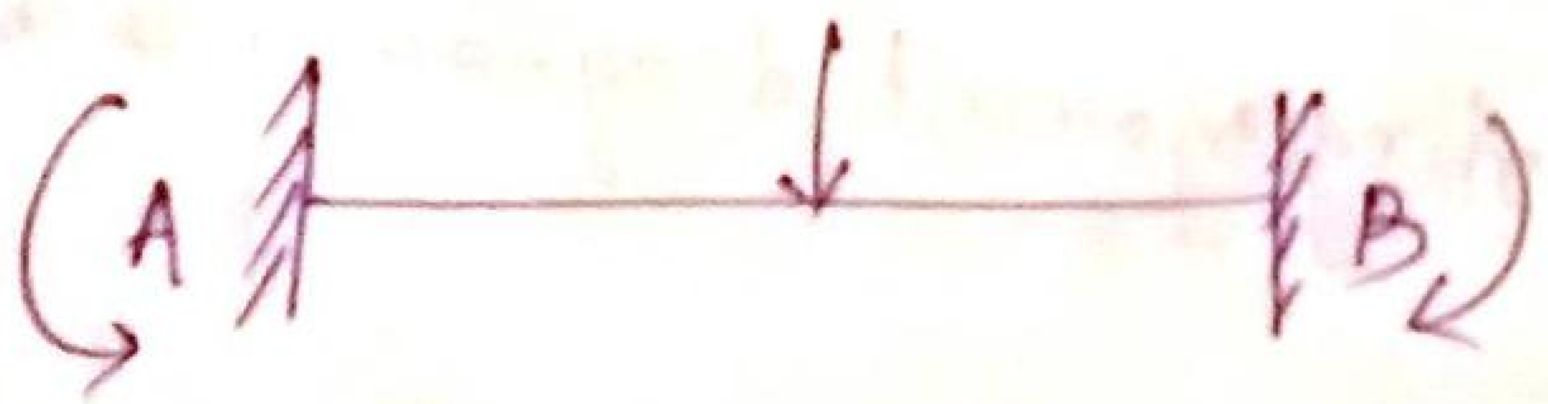
⇒ 

$$\Rightarrow \frac{1}{2} \times \frac{WL}{4} \times L = L \times M_{FAB}$$

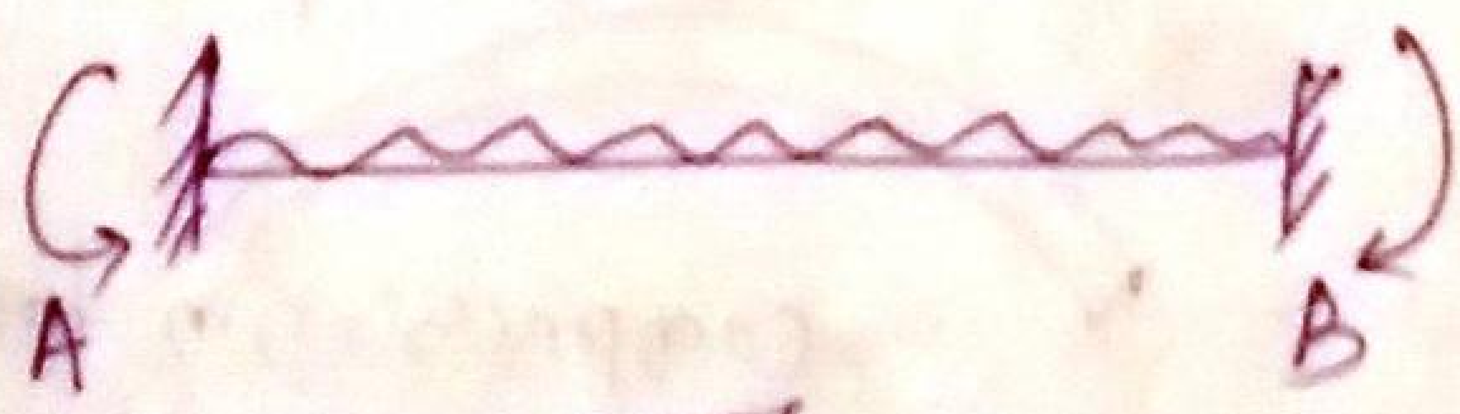
$$\Rightarrow \boxed{M_{FAB} = \frac{WL}{8}}$$

−ve for anticlockwise (M_{FAB})
+ve for clockwise (M_{FBA})

Formulas of fixed end moments when a fixed beam is subjected to various Loading :-

(1) 

$\boxed{M_{FAB} = -WL/8}$, $\boxed{M_{FBA} = +WL/8}$

(2) 

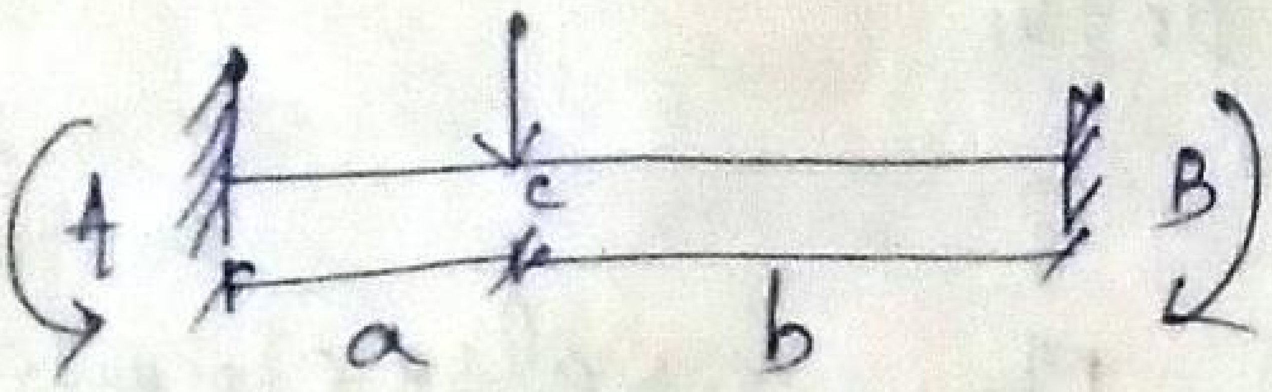
$\boxed{M_{FAB} = \frac{-WL^2}{12}}$ $\boxed{M_{FBA} = +WL^2/12}$

Note:- To calculate fixed end moment you can use any force method such as unit load method

Castigliano's theorem

Theorem of least work / min.m potential energy.

(3)

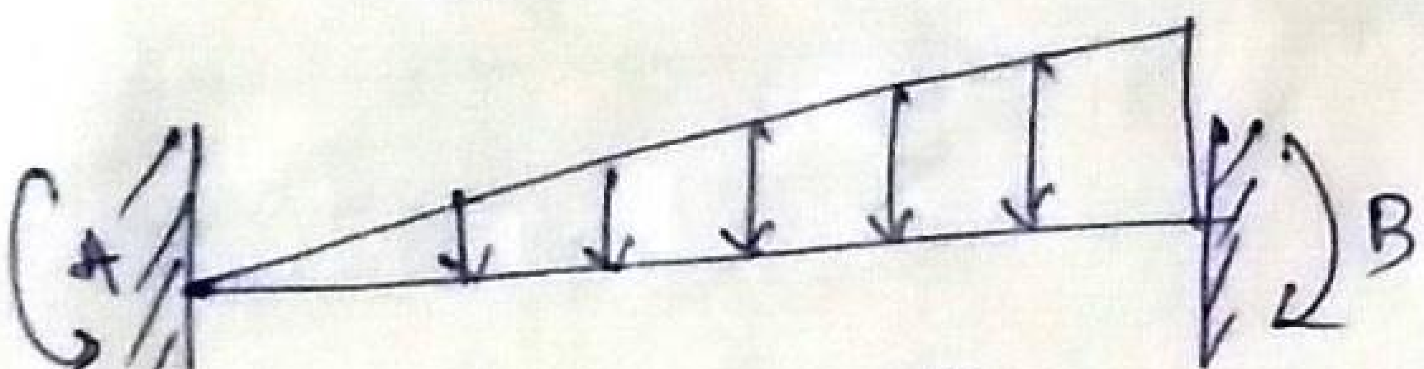


[Eccentric point loading]

$$\boxed{M_{FAB} = -\frac{wab^2}{L^2}}, \quad \boxed{M_{FBA} = +\frac{wa^2b}{L^2}}$$

Note:- The fixed end moment for eccentric loading are n't same for both support where as fixed end moment for concentric loading are same.

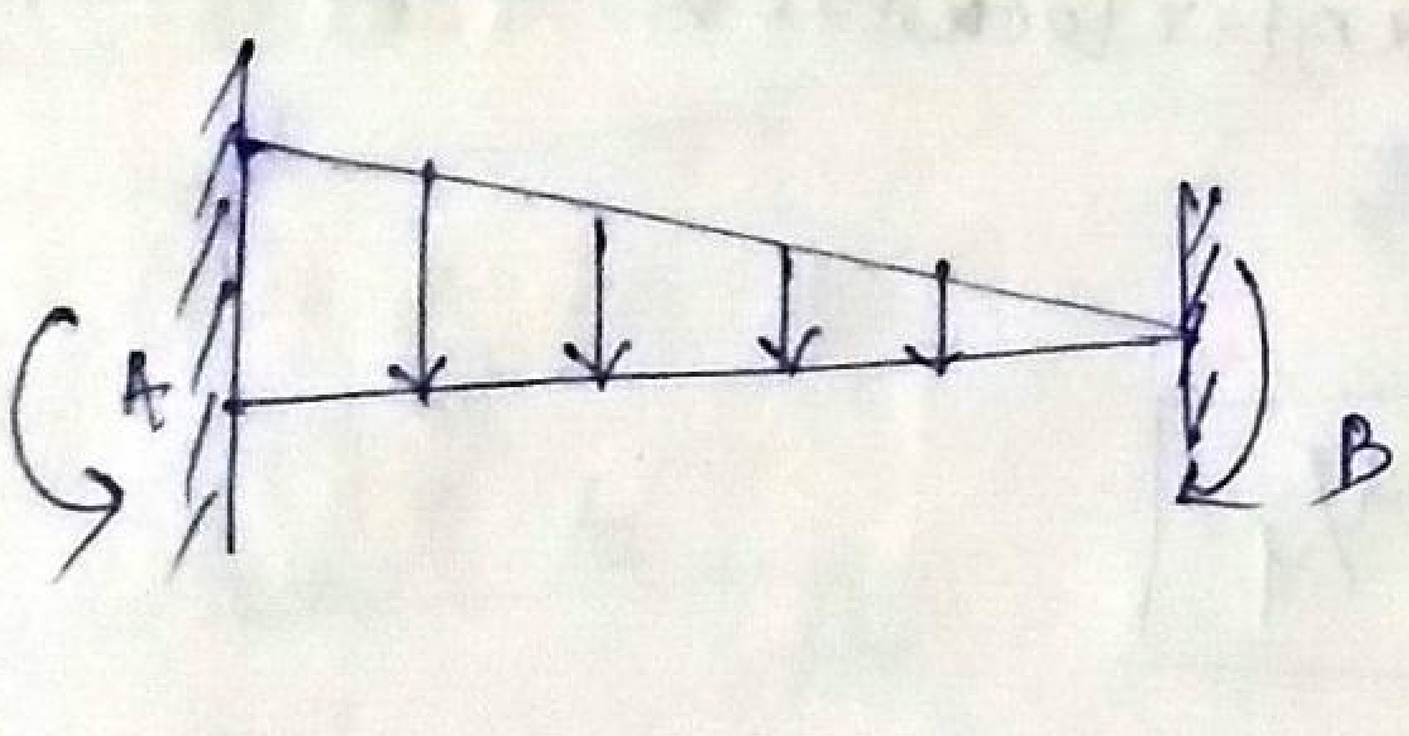
(4) uniformly varying load



$$\boxed{M_{FAB} = -\frac{wL^2}{30}}$$

$$\boxed{M_{FBA} = +\frac{wL^2}{20}}$$

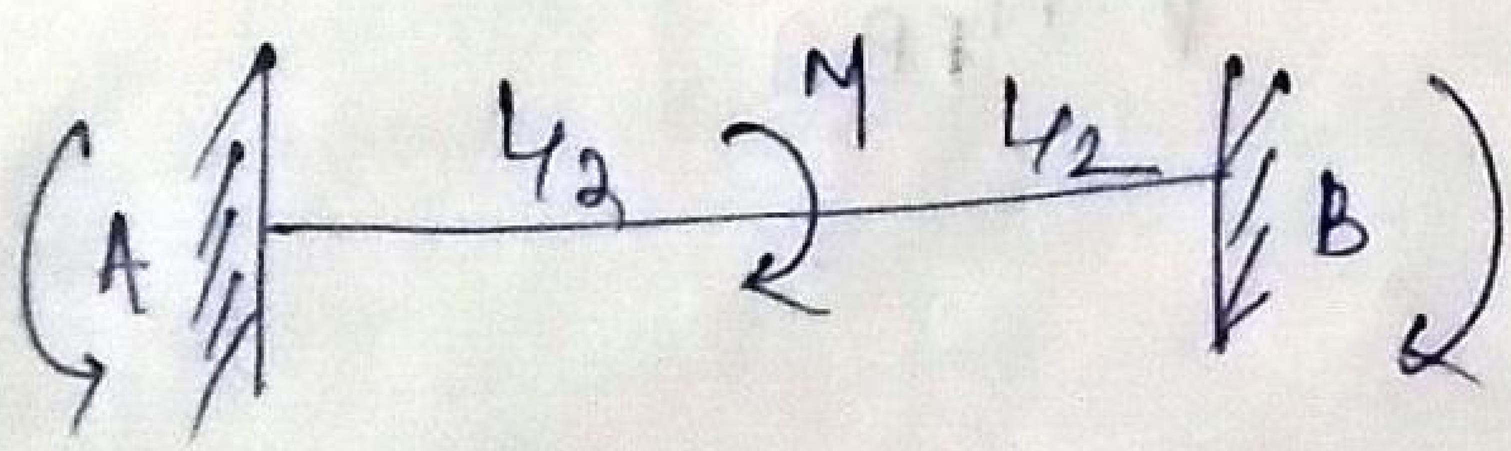
(5)



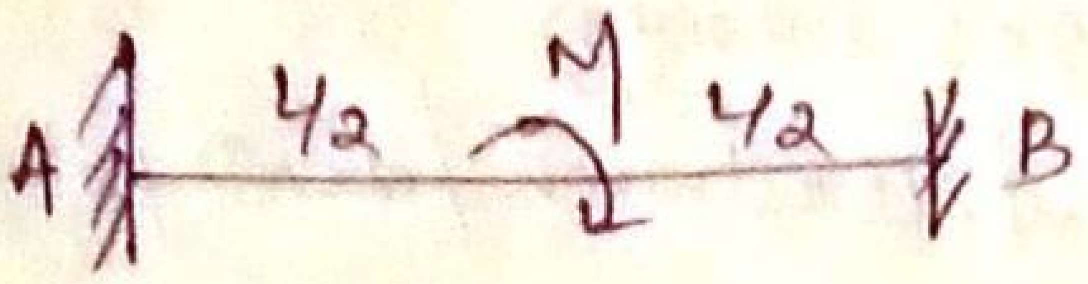
$$\boxed{M_{FAB} = -\frac{wL^2}{20}}$$

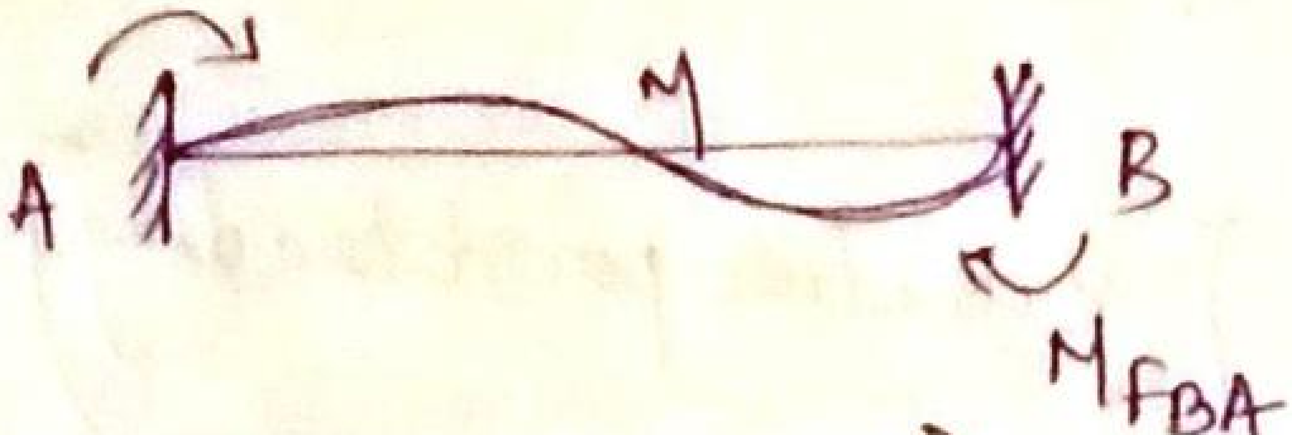
$$\boxed{M_{FBA} = +wL^2/30}$$

(6)



If a fixed beam is subjected to a concentric couple (M)





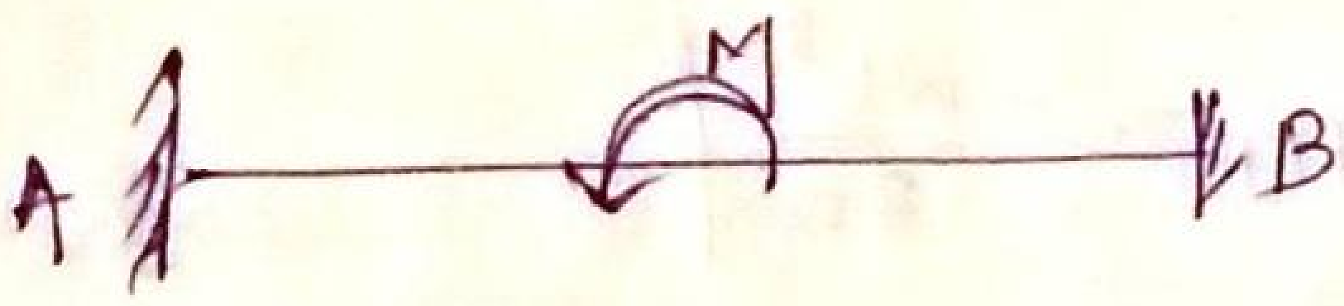
Defle

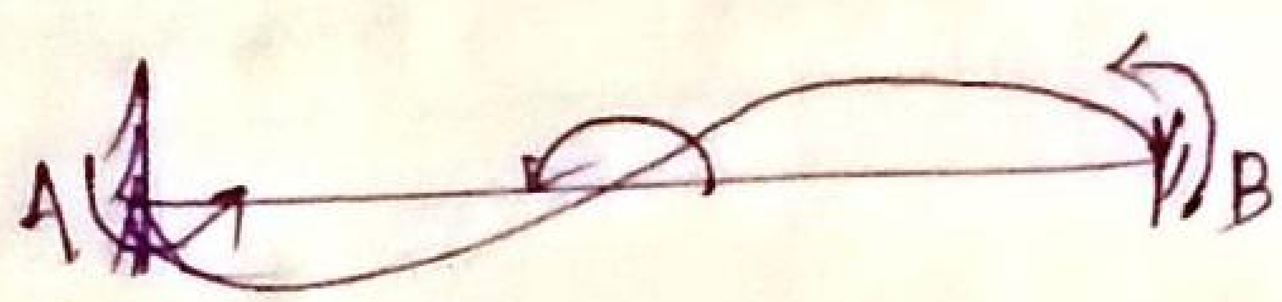
M_{FBA} is clockwise (+ve) and M_{FAB} is also clockwise (+ve).

[In this case both the moments are taken as (+ve) for a concentric couple.]

$$\boxed{M_{FAB} = M_{FBA} = M/4}$$ both sign will be same.

(7) If we consider a beam with concentric anticlockwise couple (M)

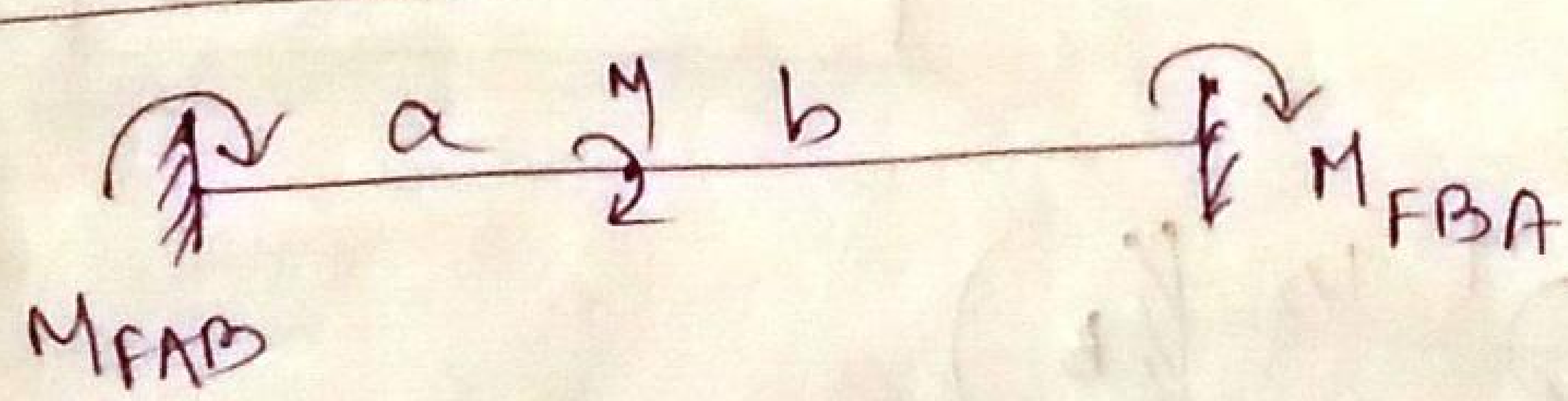




M_{FAB} and M_{FBA} both are anticlockwise and both moment are −ve.

So $$\boxed{M_{FAB} = M_{FBA} = -M/4}$$

(8)



$$\boxed{M_{FAB} = \frac{Mb}{L^2}(2a-b)}, \quad \boxed{M^F_{BA} = \frac{Ma}{L^2}(2b-a)}.$$

* Then formula of fixed end moment for different types of loading will used for different analysis like

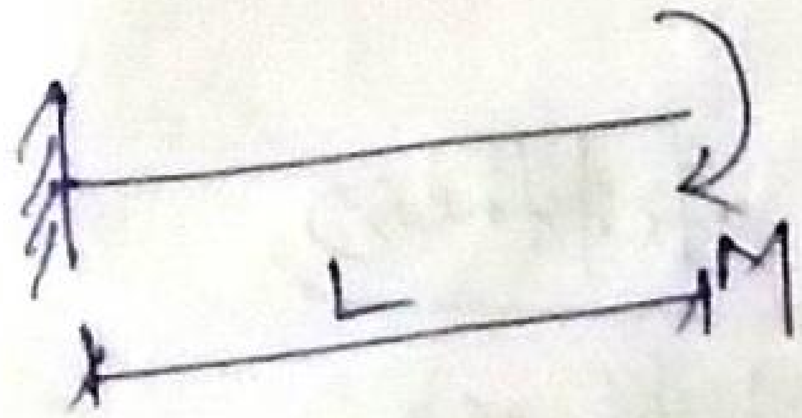
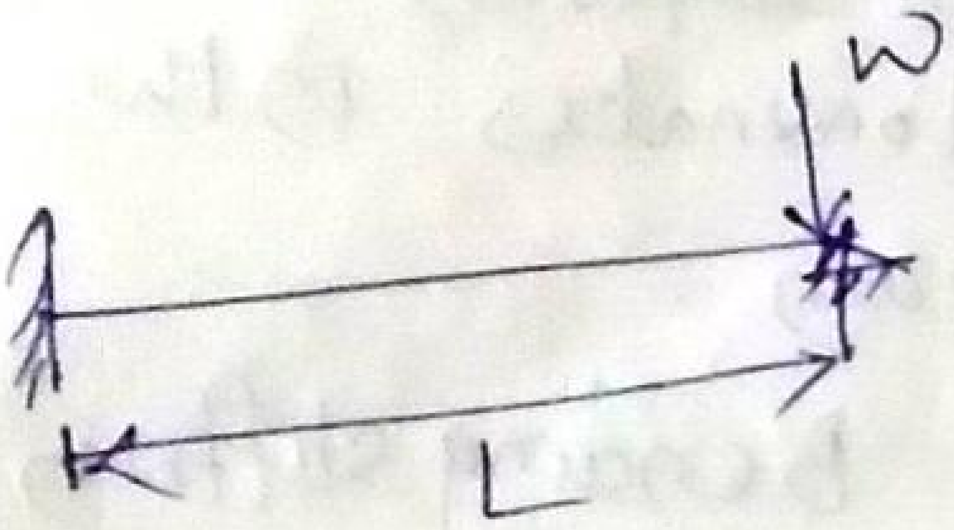
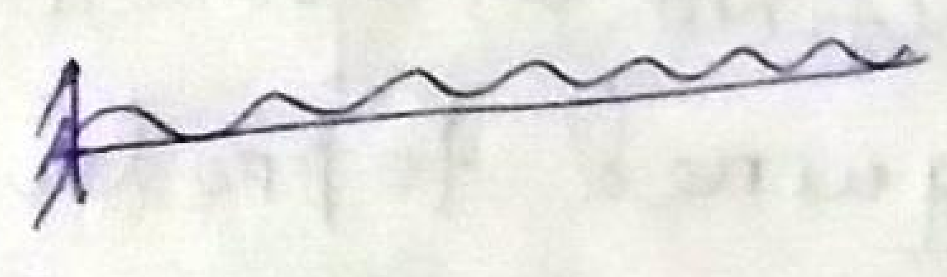
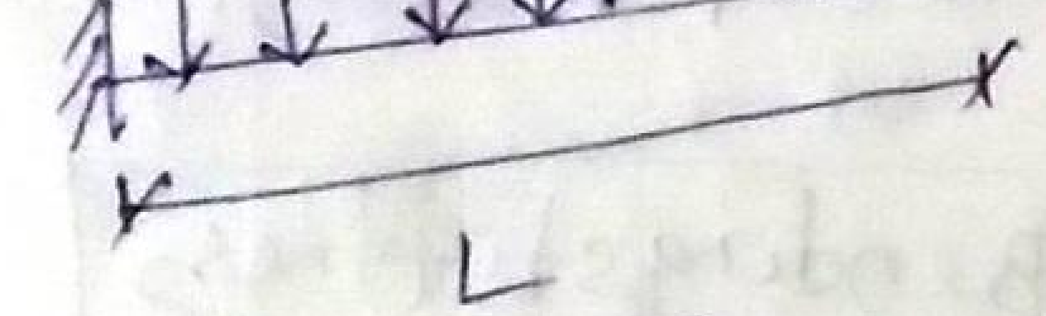
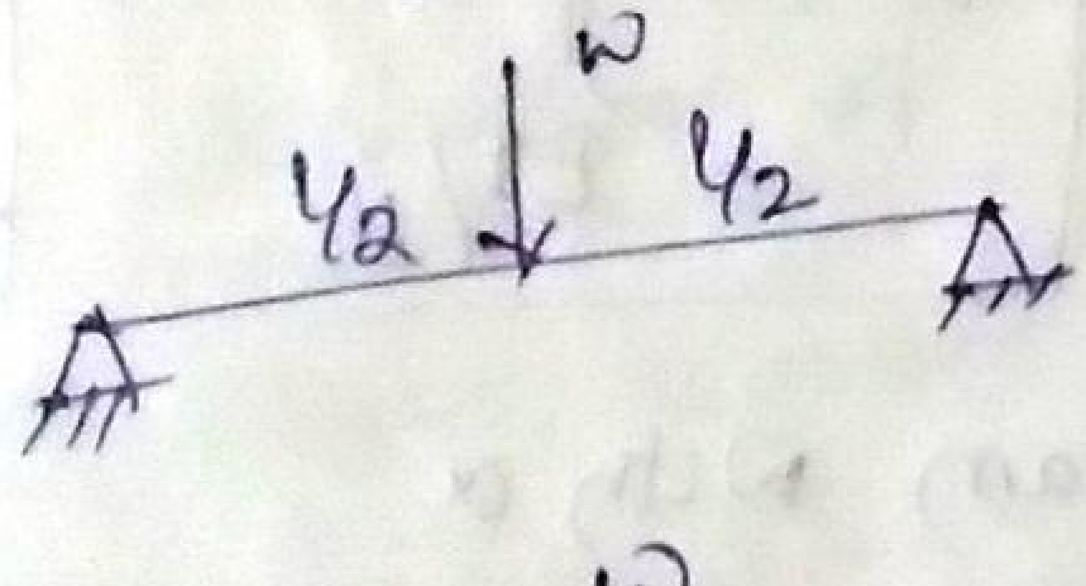
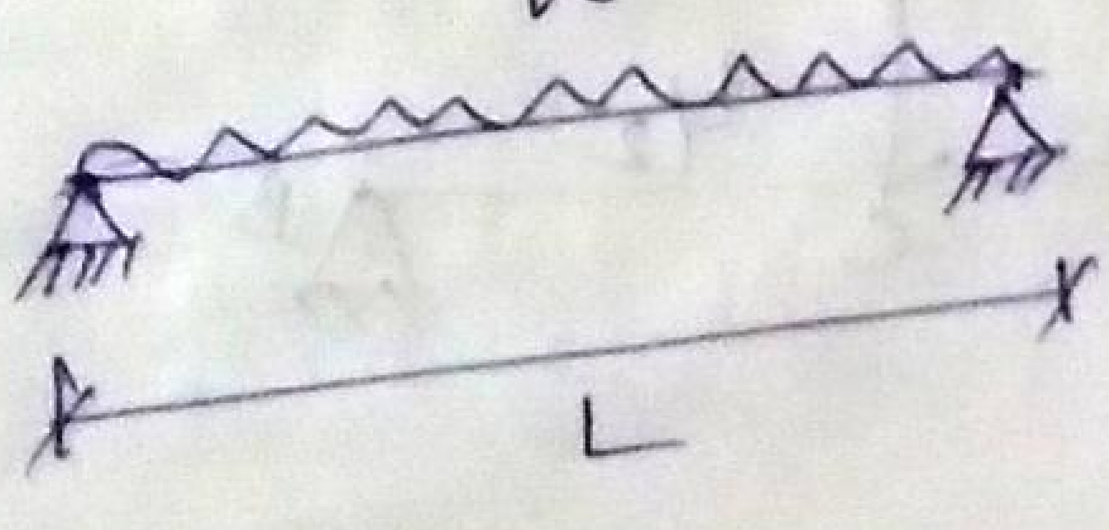
Slope deflection method

Moment distribution method

Stiffness method

Displacement method.

Shortcut formula:-

Type of beam	Maximum B.M	Slope	Deflection
(1) 	M	$\theta = \frac{ML}{EI} = \frac{ML}{EI}$	$\delta = \theta \times \frac{L}{2} = \frac{ML^2}{2EI}$
(2) 	WL	$\theta = \frac{ML}{2EI} = \frac{WL^2}{2EI}$	$\delta = \theta \times \frac{2L}{3} = \frac{WL^3}{3EI}$
(3) 	$\frac{WL^2}{2}$	$\theta = \frac{ML}{3EI} = \frac{WL^3}{6EI}$	$\left(\frac{WL^4}{8EI}\right)$
(4) 	$\frac{WL^2}{6}$	$\theta = \frac{ML}{4EI} = \frac{WL^3}{24EI}$	$\left(\frac{WL^4}{30EI}\right)$
(5) 	$\frac{WL}{4}$	$\theta = \frac{ML}{4EI} = \frac{WL^2}{16EI}$	$\left(\frac{WL^3}{48EI}\right)$
(6) 	$\frac{WL^2}{8}$	$\theta = \frac{ML}{3EI} = \frac{WL^3}{24EI}$	$\left(\frac{5WL^4}{384EI}\right)$

Stiffness of material

Stiffness can be defined as force to unit displacement

→ More stiff material it means more force required to produce unit displacement

Similarly less stiff material it means less force required to produce unit displacement.

→ So stiffness is the ability of the material to resist the deformation, displacement or deflection.

There are 2 types of stiffness

(1) Axial stiffness

(2) Bending stiffness.

Axial stiffness

This stiffness predominate in the trusses

→ Axial stiffness is defined as the force required to produce unit displacement

→ The force may be compressive or tensile.

$$\boxed{\text{Axial stiffness} = \frac{EA}{L}}$$

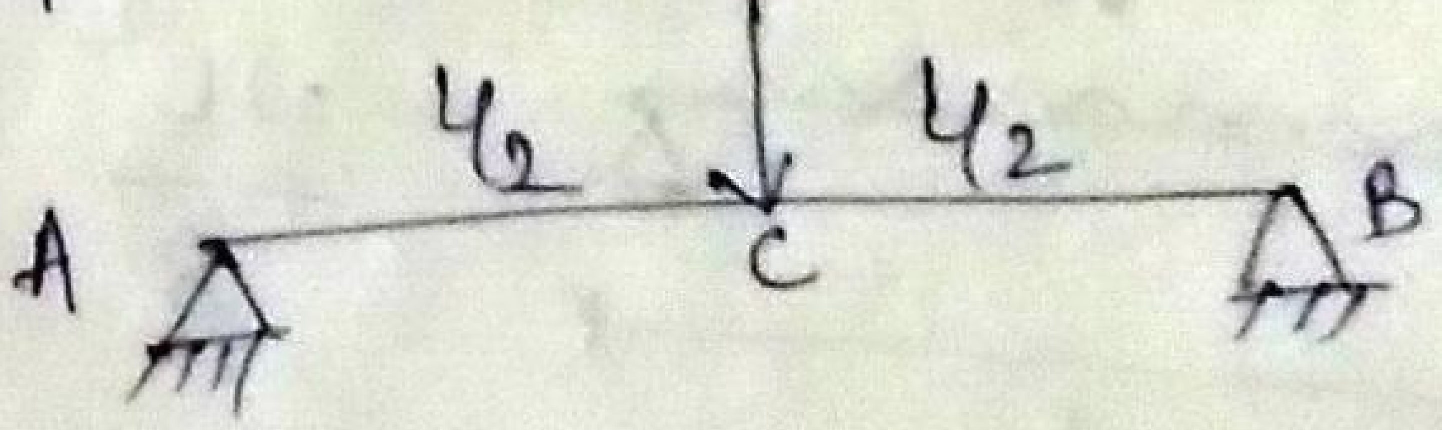
Bending stiffness

This stiffness predominates in the beam.

→ Bending stiffness is defined as moment required to produce unit rotation.

$$\boxed{\text{Bending stiffness} = EI/L}$$

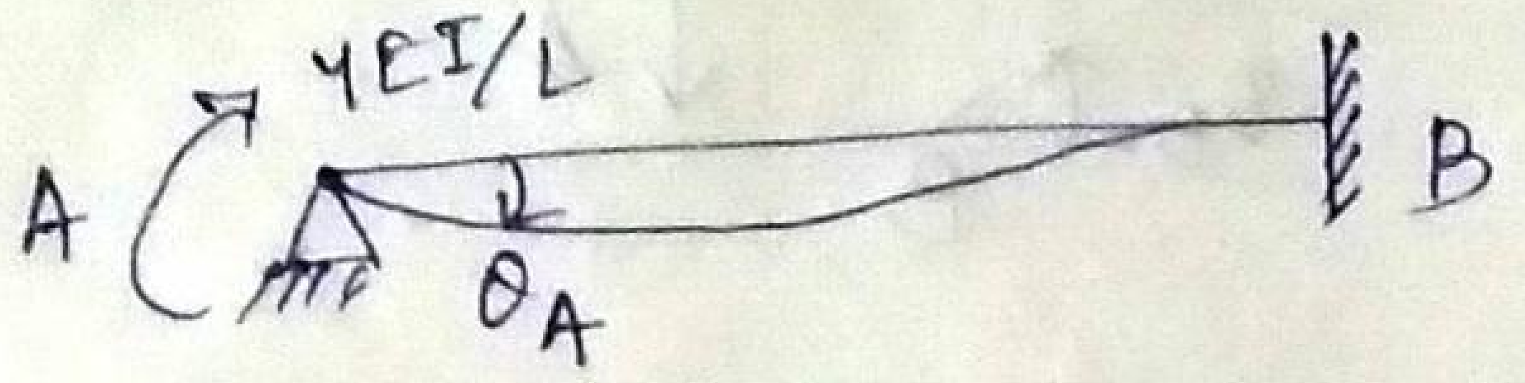
* Consider a simply supported beam with a point load at centre



As this beam is simply supported and concentric then Rotation developed at A and B will be

$$\theta_A = \theta_B = \frac{ML}{4EI}.$$

* If we replace a fixed support instead of hinge support



Rotation $\theta = \frac{ML}{4EI}$

$\Rightarrow 4EI\theta = ML$

$\Rightarrow M = \frac{4EI}{L}\theta$

We k/w stiffness $\boxed{K = M/\theta_A}$ $\Rightarrow K = \frac{\frac{4EI}{L} \times \cancel{\theta_A}}{\cancel{\theta_A}}$

$\Rightarrow \boxed{K = 4EI/L}$.

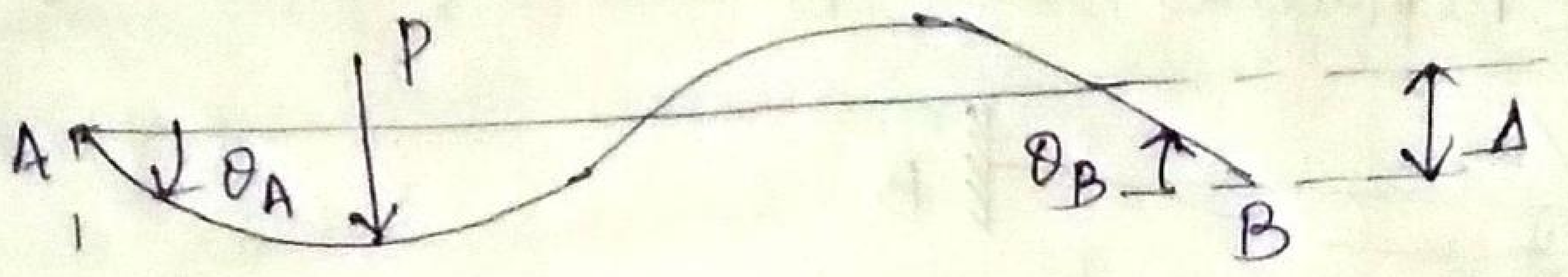
* for hinged support stiffness $K = 4EI/L$ and moment is $\frac{4EI}{L}\theta_A$. This moment will be carry over to the fixed support as $2EI/L\ \theta_A$.

* <u>Derivation of slope deflection equation</u>

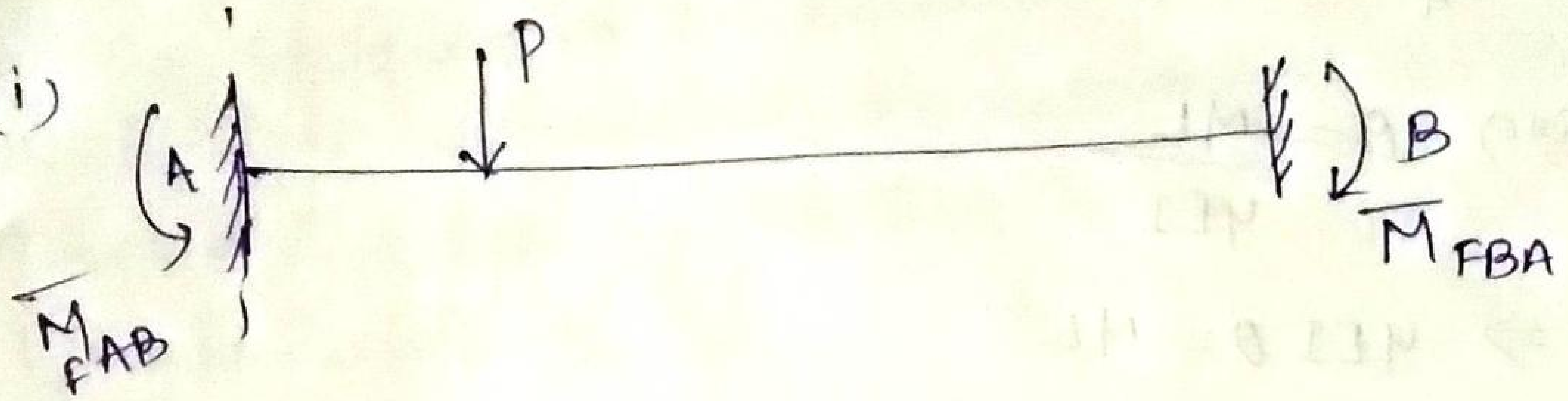
Consider a segment AB of a continuous beam as shown in figure. The slope deflection equation are derived by superimposing the end moments developed due to

(i) Applied Load (ii) Rotation of joint A (θ_A)

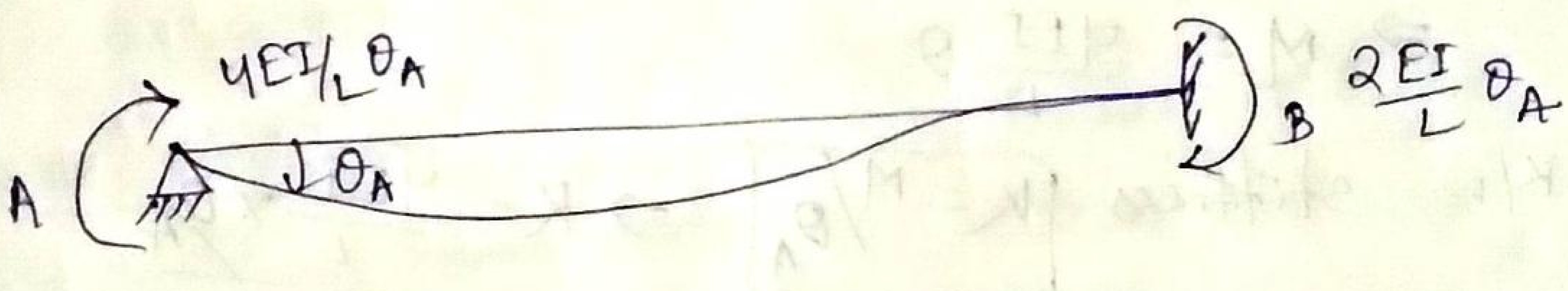
(iii) Rotation of joint B (θ_B) (iv) settlement of support B w.r.t support A (Δ).



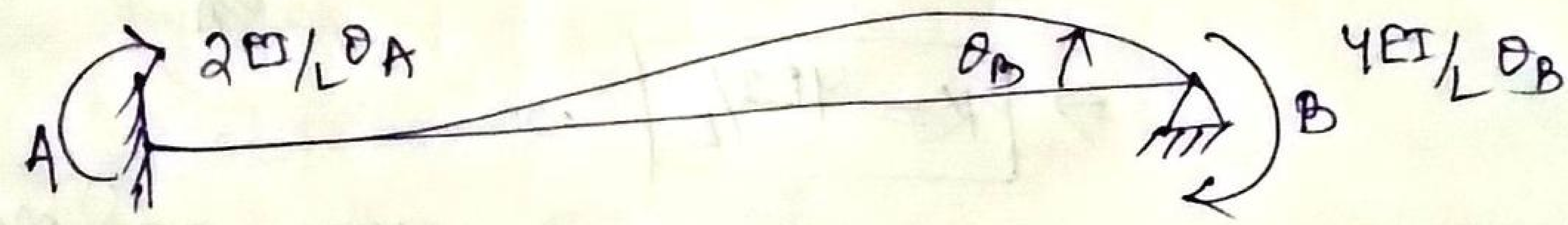
(i)



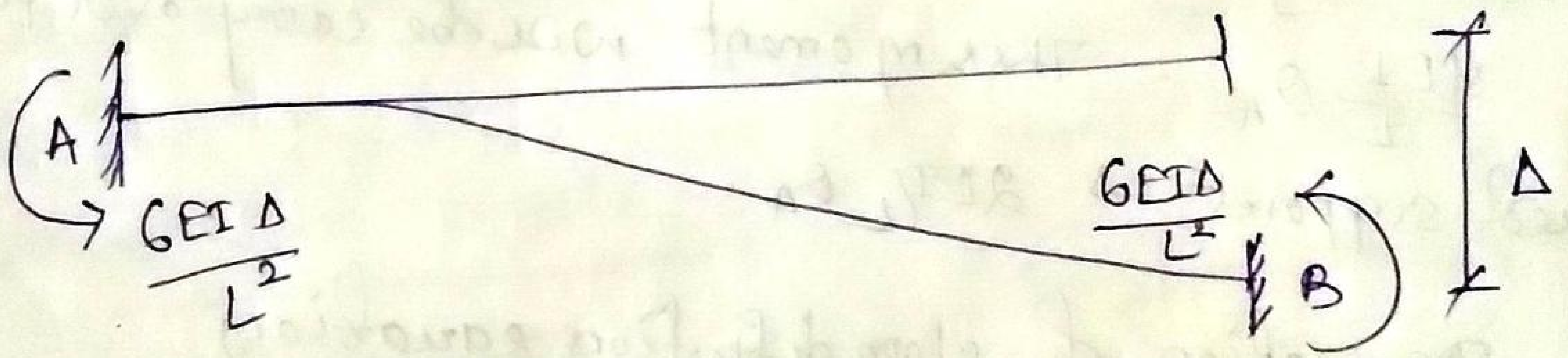
(ii)



(iii)



(iv)



The span AB can be subjected to combination of multiple Loading as shown

(i) In beam shown in figure (i) the fixed end moments at ends A and B will be $\overline{M}_{AB}$ and $\overline{M}_{BA}$ respectively.

(II) In beam AB shown in figure (II) if joint A rotates by angle θ_A, then fixing moments at end A and B will be

$$\overline{M}_{AB} = \frac{4EI}{L}\theta_A , \quad \overline{M}_{BA} = \frac{2EI}{L}\theta_A .$$

(III) In beam shown in figure (III) if joint B rotates by angle θ_B then fixing moments at end B and A will be

$$\overline{M}_{BA} = \frac{4EI}{L}\theta_B , \quad \overline{M}_{AB} = \frac{2EI}{L}\theta_B .$$

(IV) If support 'B' settles down 'Δ' with respect to support A causing rotation to member BA in clockwise direction, then fixing moment produce at B and A will be

$$\overline{M}_{BA} = -6EI\Delta/L^2$$

$$\overline{M}_{AB} = -6EI\Delta/L^2 .$$

The final moment at end A and B due to multiple effect

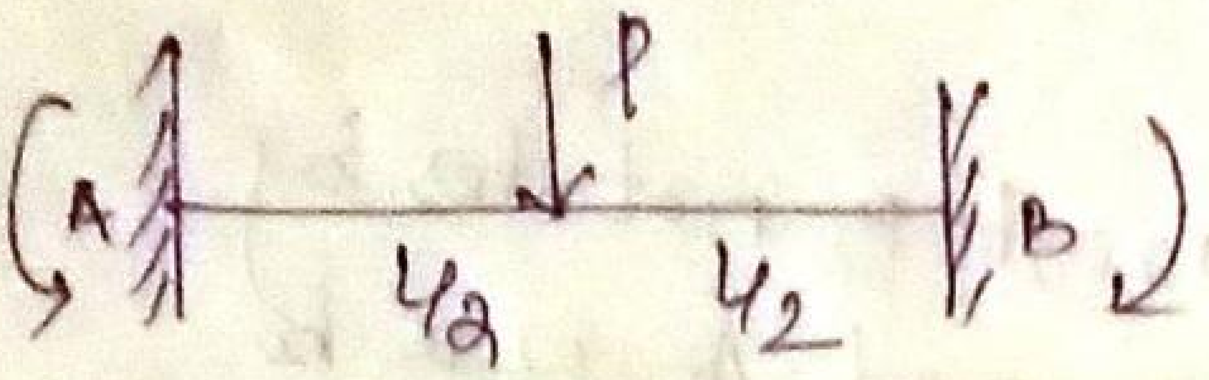
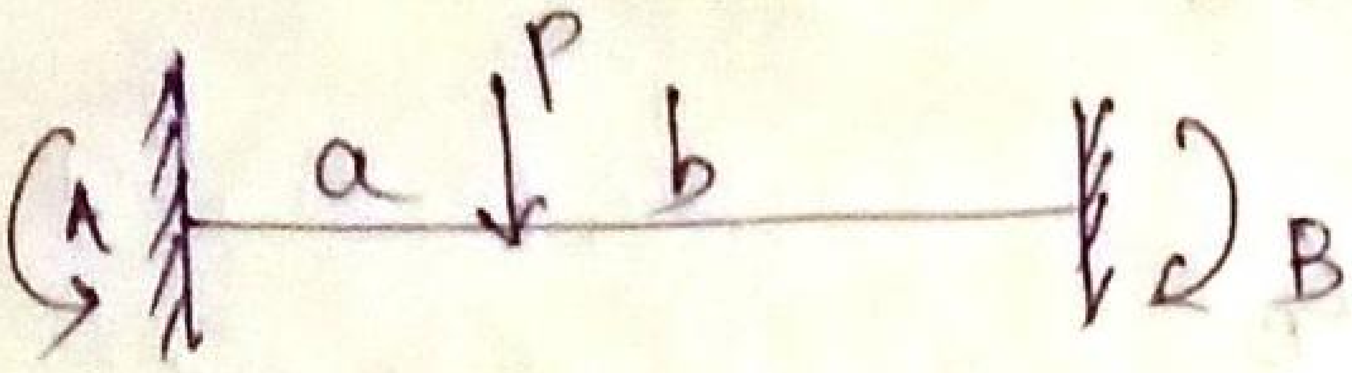
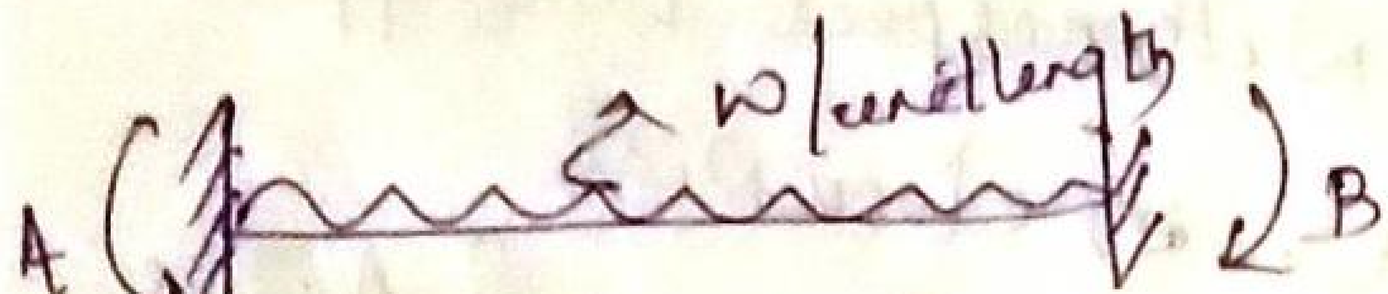
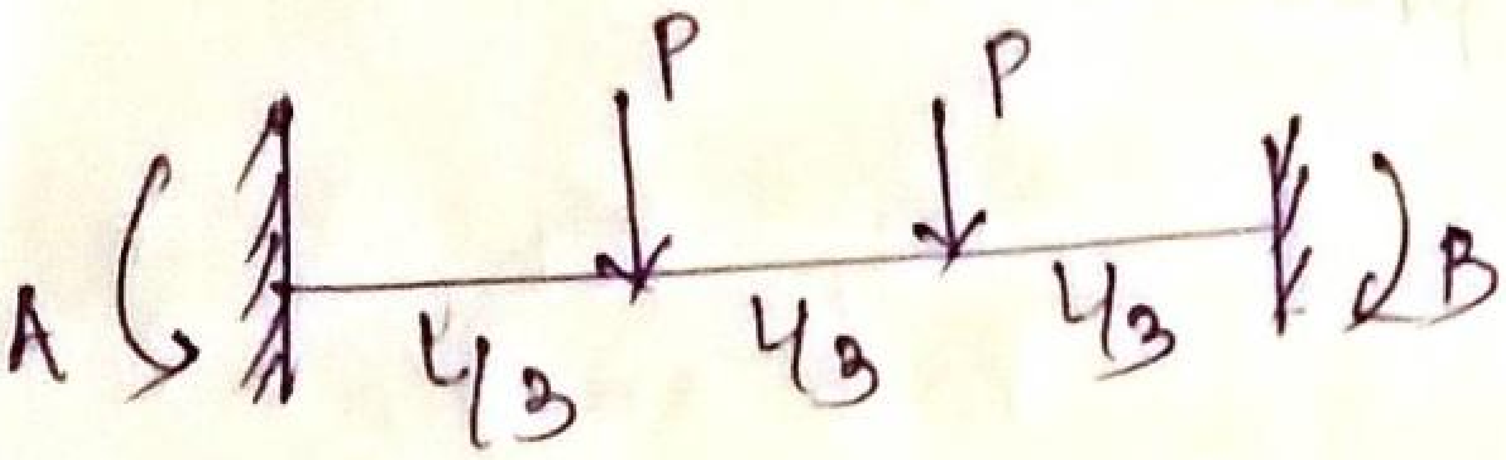
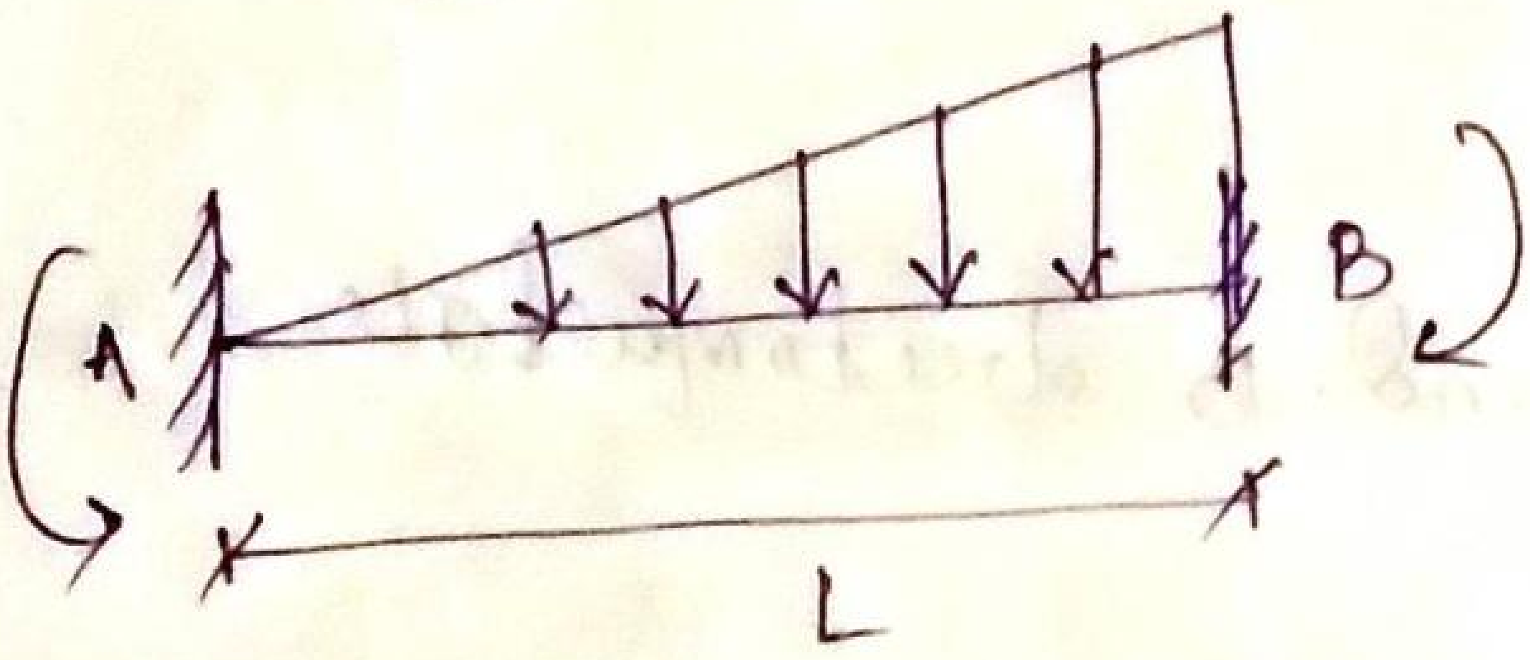
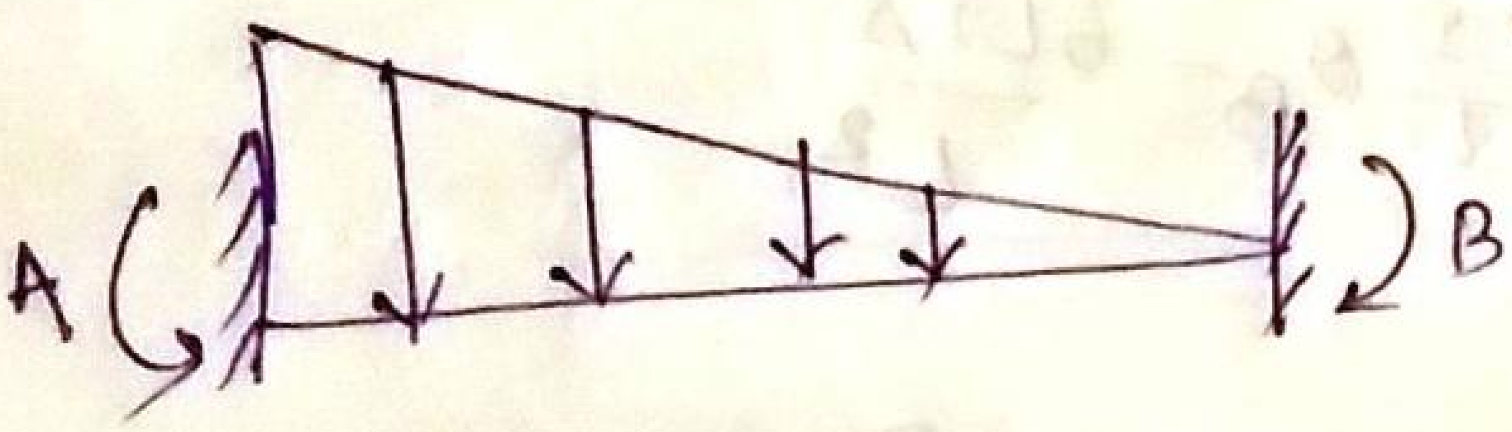
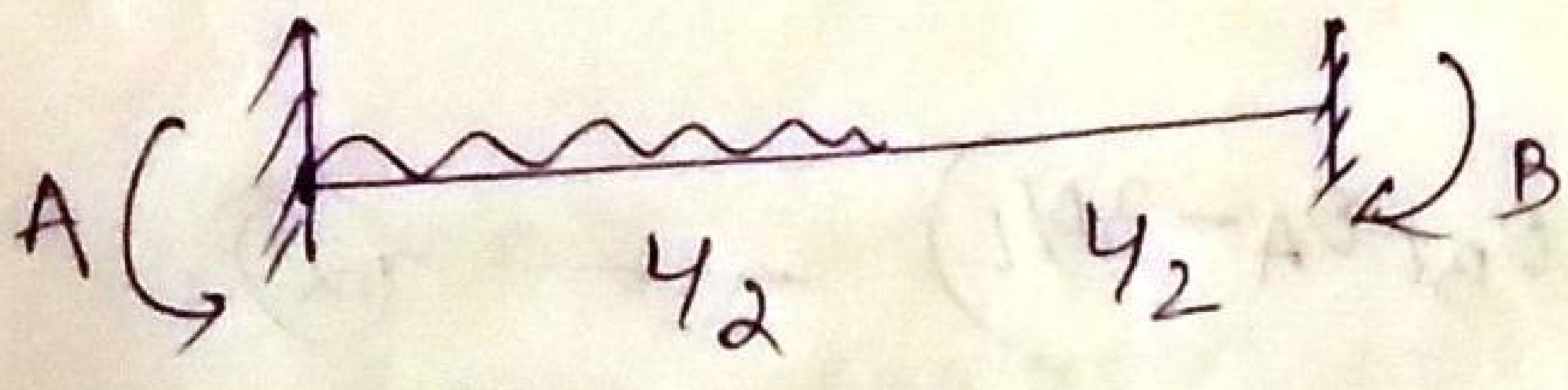
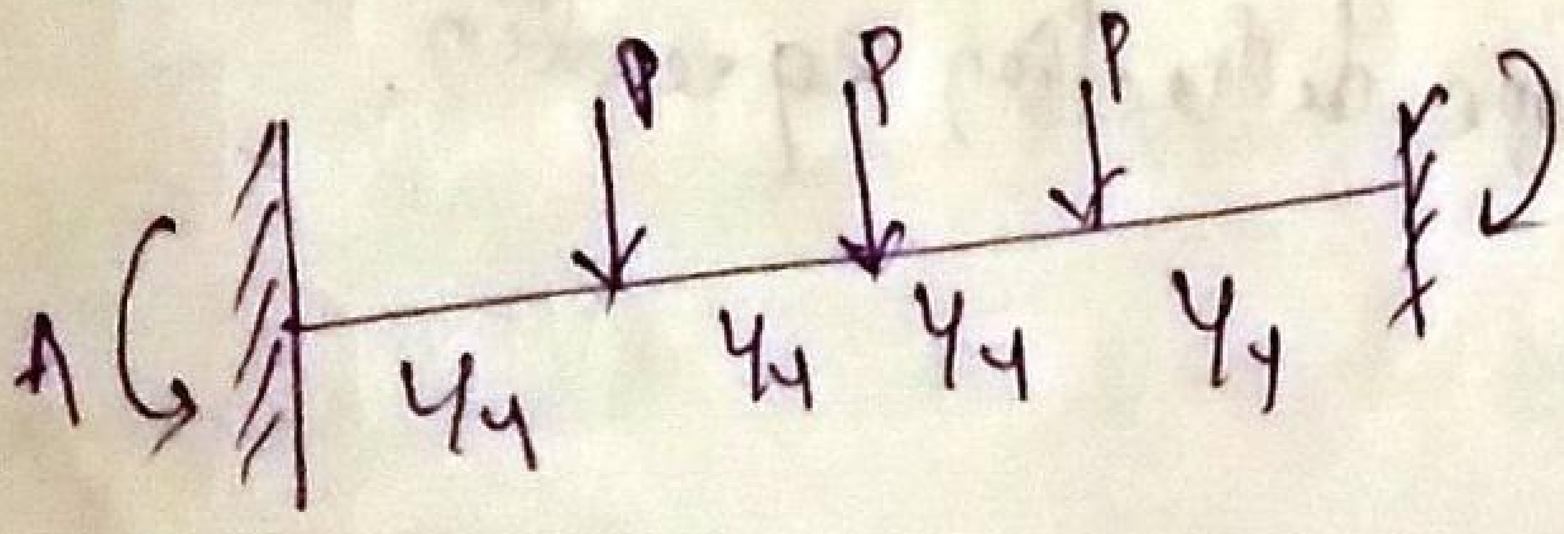
$$M_{AB} = \overline{M}_{AB} + \frac{4EI}{L}\theta_A + \frac{2EI}{L}\theta_B - \frac{6EI\Delta}{L^2}$$

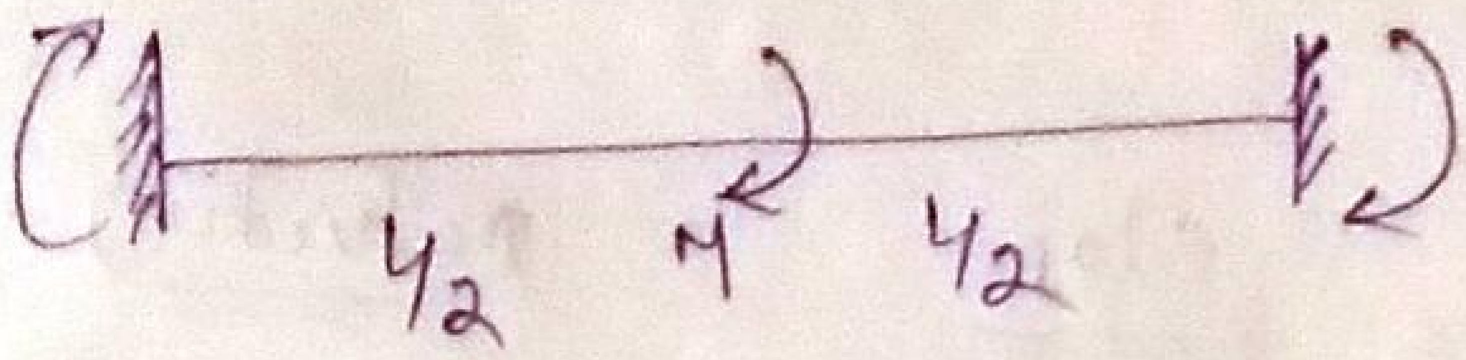
$$M_{AB} = \overline{M}_{AB} + \frac{2EI}{L}\left(2\theta_A + \theta_B - 3\Delta/L\right) \quad \text{——— (1)}$$

$$M_{BA} = \overline{M}_{BA} + \frac{2EI}{L}\left(2\theta_B + \theta_A - 3\Delta/L\right) \quad \text{——— (2)}$$

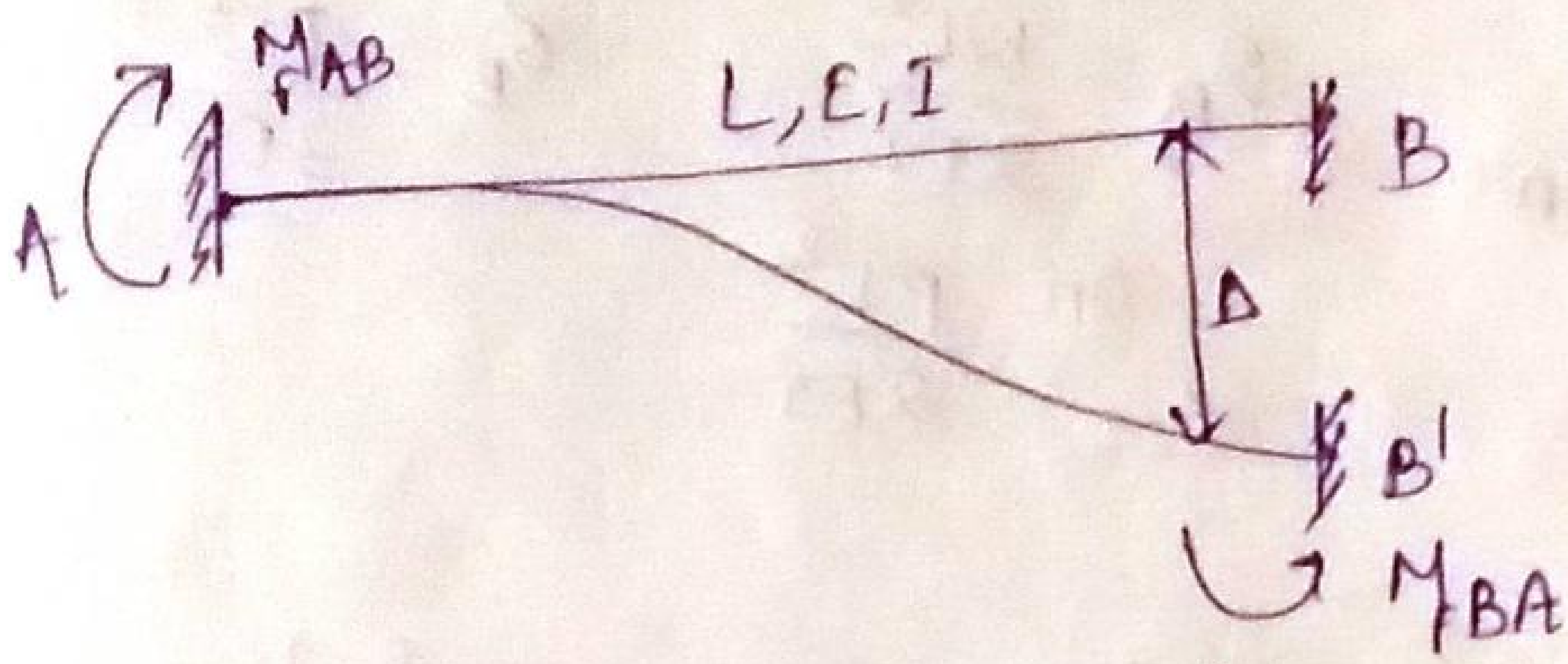
Equation (1) and (2) are slope deflection equation.

Fixed end Moment

Beam	F_{AB}	F_{BA}
	$-\frac{WL}{8}$	$+\frac{WL}{8}$
	$-\frac{wab^2}{L^2}$	$+\frac{wa^2b}{L^2}$
	$-\frac{wL^2}{12}$	$+\frac{wL^2}{12}$
	$-\frac{2PL}{9}$	$+\frac{2PL}{9}$
	$-\frac{wL^2}{30}$	$+\frac{wL^2}{20}$
	$-\frac{wL^2}{20}$	$+\frac{wL^2}{30}$
	$-\frac{11}{192}wL^2$	$-\frac{5}{192}wL^2$
	$-\frac{15PL}{48}$	$+\frac{15PL}{48}$



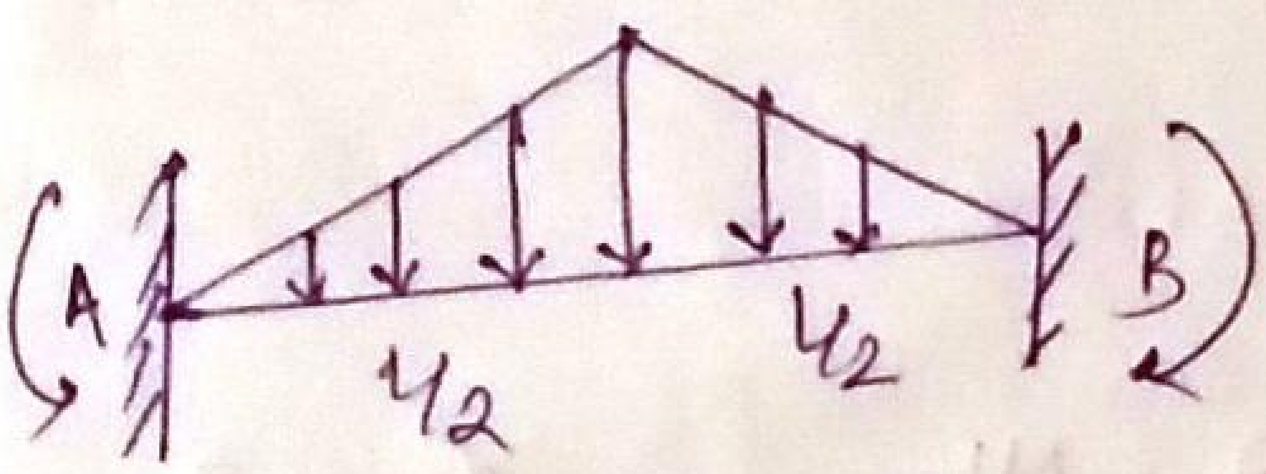
$M_{FAB} = M_{FBA} = +M/4$



$M_{FAB} = -6EI\Delta/L^2$

$M_{FBA} = -6EI\Delta/L^2$.

Note:- If settlement of support causes rotation of member in clockwise direction then fixed end moment developed will be in anticlockwise direction or vice versa.

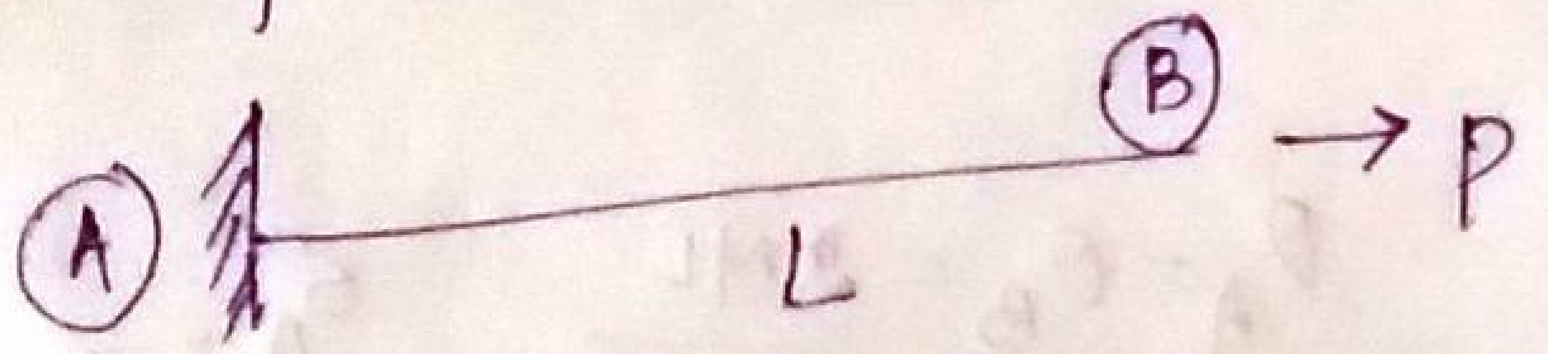


$M_{FAB} = -\frac{5wL^2}{96}$

$M_{FBA} = +5wL^2/96$.

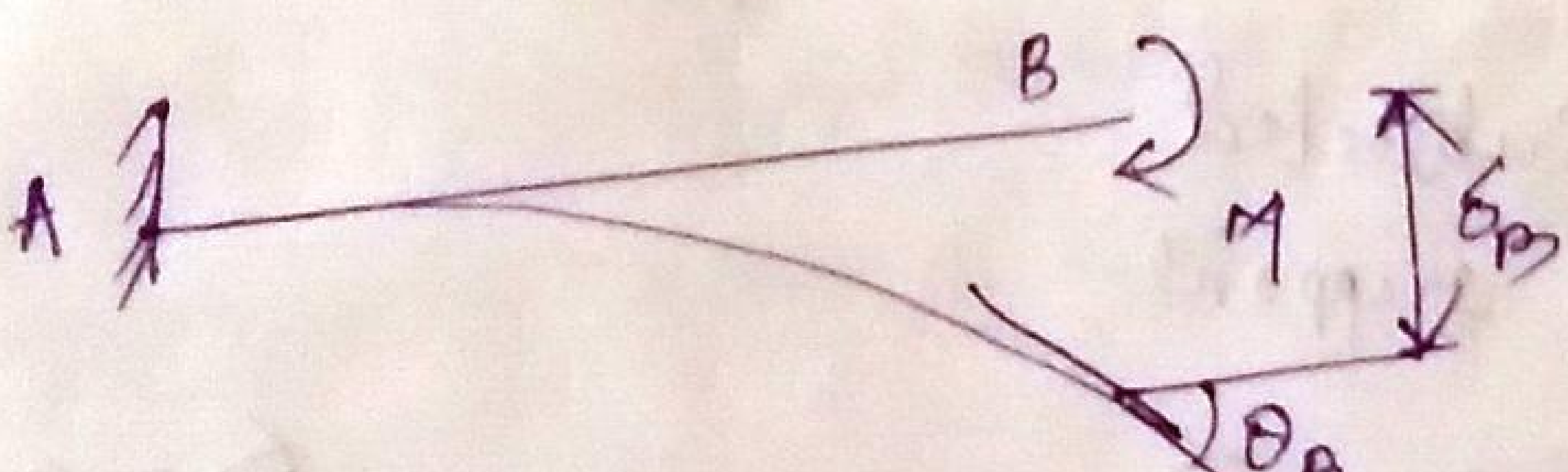
Standard value of slope and deflection

(1) Axial Load at free end of cantilever.



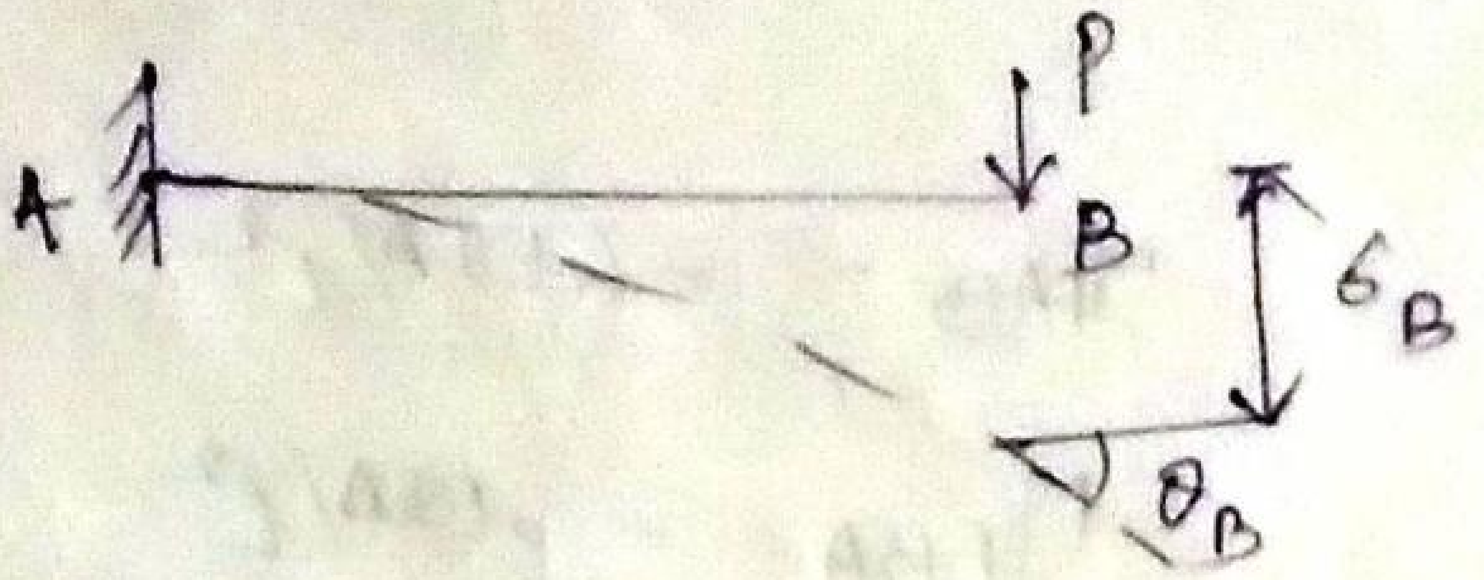
Slope	Deflection
$\theta_B = 0$	$\delta_B = \frac{PL}{AE}$

(2) Moment at free end of cantilever



$\theta_B = \frac{ML}{EI}$ $\delta_B = \frac{ML^2}{2EI}$

(3) point load at free end of cantilever.



Slope

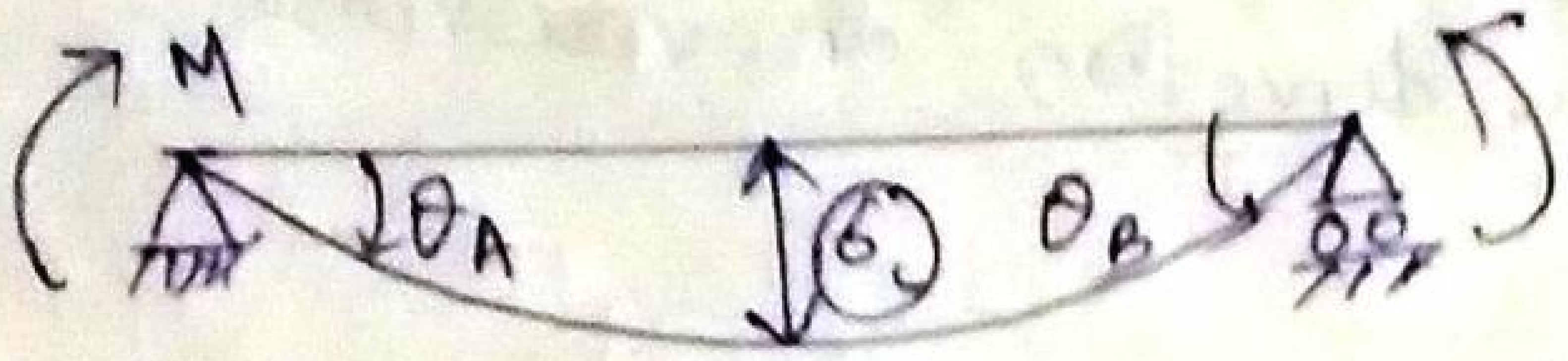
$$\theta_B = \frac{WL^2}{2EI}$$

or $\frac{PL^2}{2EI}$

Deflection

$$\delta_B = \frac{PL^3}{3EI}$$

(4) Simply supported beam with moment at both end.



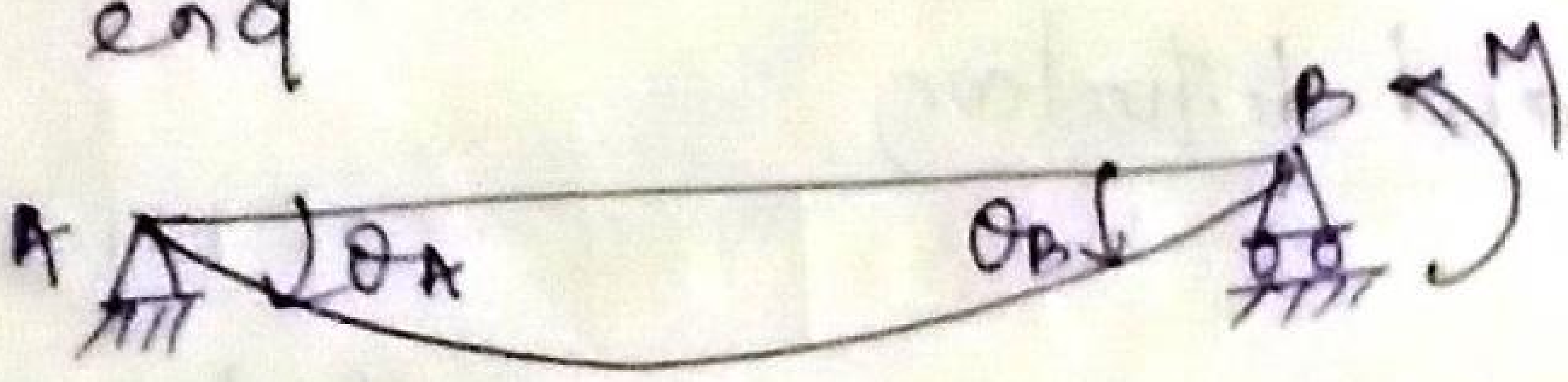
Slope

$$\theta_A = \theta_B = \frac{ML}{2EI}$$

Deflection

$$\delta_c = \frac{ML^2}{8EI}$$

(5) Simply supported beam with moment at one end



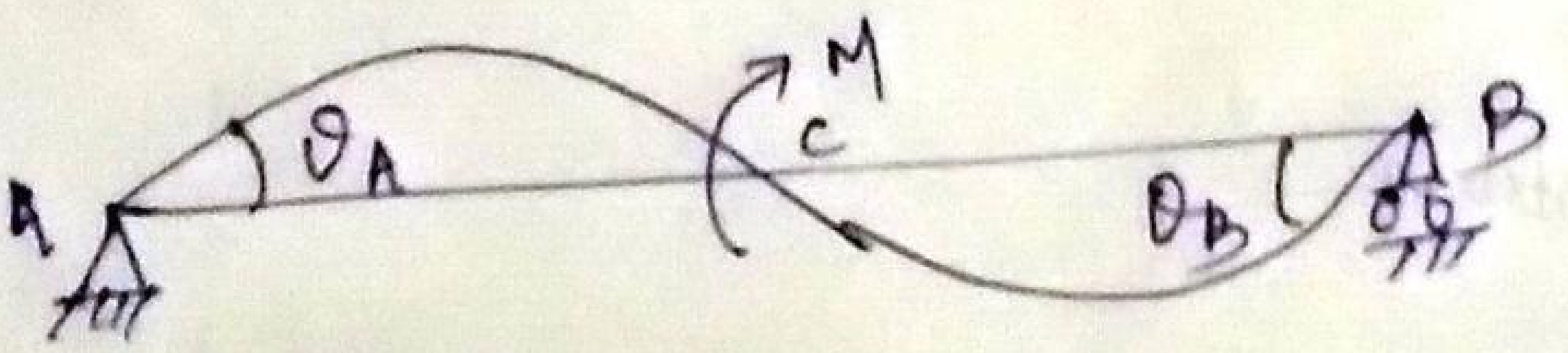
$$\theta_A = \frac{ML}{6EI}$$

$$\theta_B = \frac{ML}{3EI}$$

$\delta_A = 0$

$\delta_B = 0$

(6) Simply supported beam with moment at midspan



$$\theta_A = \theta_B = \frac{ML}{24EI}$$

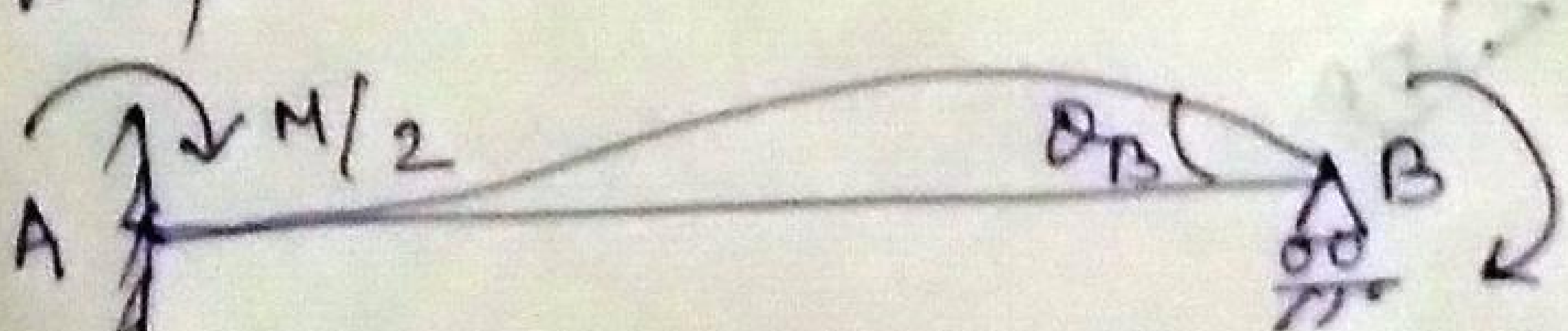
$$\theta_c = \frac{ML}{12EI}$$

$\delta_A = 0$

$\delta_B = 0$

$\delta_c = 0$

(7) propped cantilever subjected to moment at propped support



$$\theta_B = \frac{ML}{4EI}$$

$\delta_A = 0$

$\delta_B = 0$

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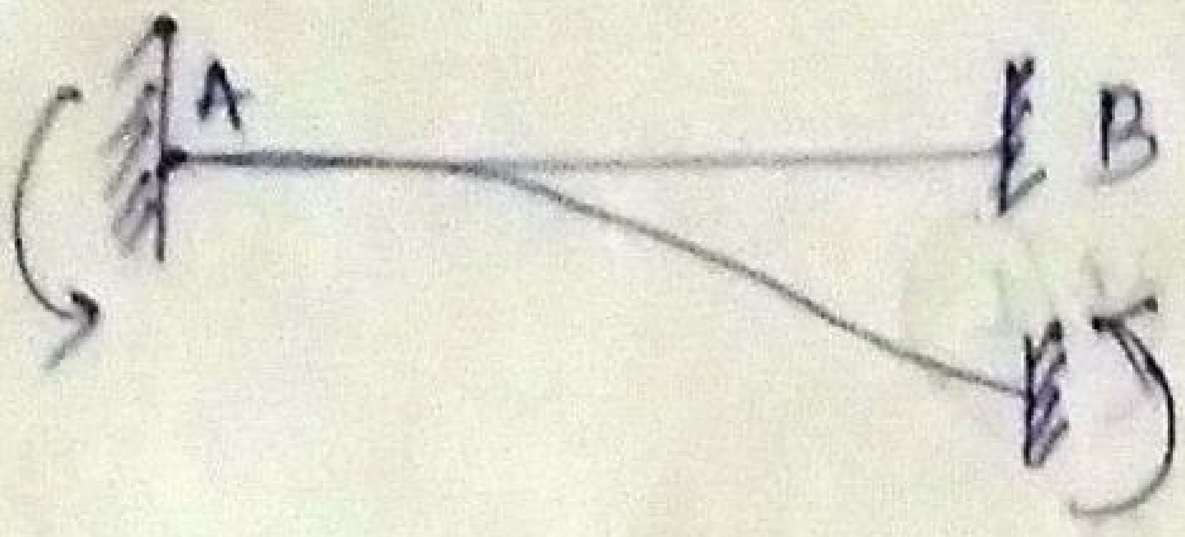
8)

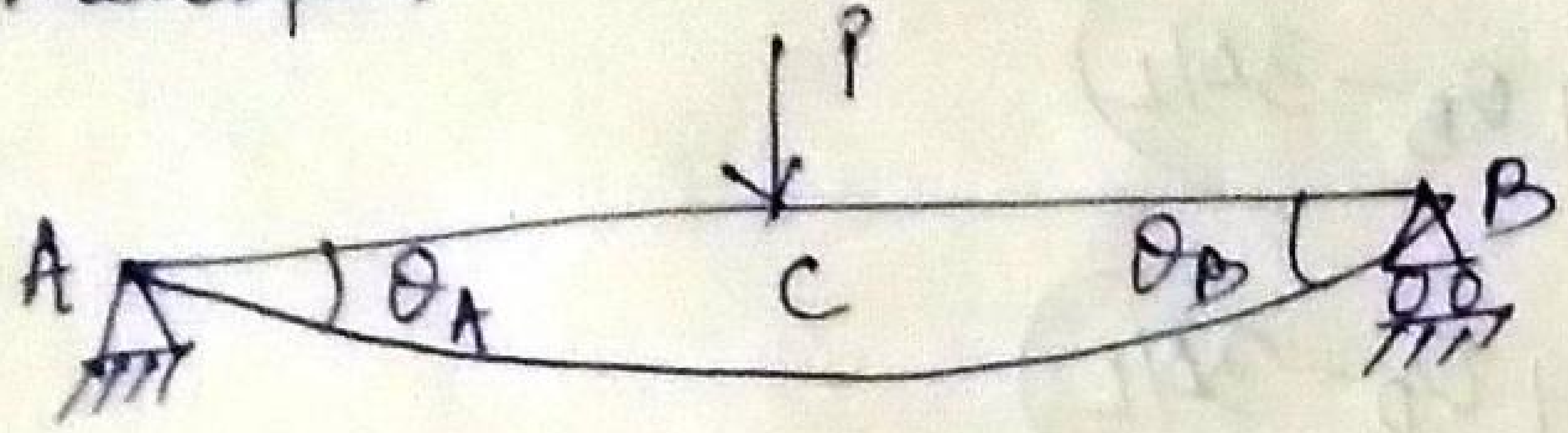


$M_{AB} = M_{BA} = 6EI\Delta/L^2$

$\delta_A = 0$

$\delta_B = 0$.

(9) Simply supported beam subjected to point load at midspan.



$\theta_A = \theta_B = \frac{WL^2}{16EI}$

$\delta_C = \frac{WL^3}{48EI}$

$\delta_A = \delta_B = 0$.

Slope deflection Method

procedure of Analysis:-

Step-1:- consider each span fixed and find fixed end moment for each span due to given loading.

Step-2:- Take θ and Δ as unknown and write slope deflection equation for end moments

Step-3:- find joint displacement (θ & Δ) by using equilibrium condition.

Step-4:- substituting value of unknowns in slope deflection equation and determine end moments.

Step-5:- Draw final bending moment diagram by superimposing B.M.D of end and free B.M.D for each span

Slope deflection equation

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left(2\theta_A + \theta_B - 3\Delta/L\right)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}\left(2\theta_B + \theta_A - 3\Delta/L\right)$$

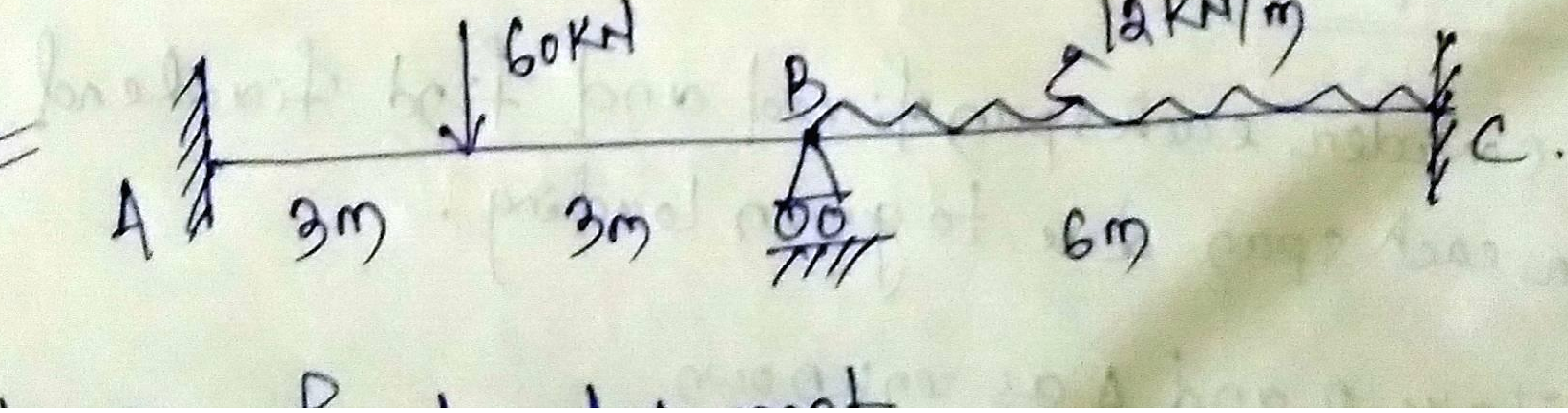
$$M_{BC} = M_{FBC} + 2EI/L\left(2\theta_B + \theta_C - 3\Delta/L\right)$$

$$M_{CB} = M_{FCB} + 2EI/L\left(2\theta_C + \theta_B - 3\Delta/L\right)$$

$$M_{CD} = M_{FCD} + 2EI/L\left(2\theta_C + \theta_D - 3\Delta/L\right)$$

$$M_{DC} = M_{FDC} + 2EI/L\left(2\theta_D + \theta_C - 3\Delta/L\right)$$

Q



Slope deflection equation

$$M_{AB} = M_{FAB} + \frac{2EI}{L}(2\theta_A + \theta_B - 3\Delta/L)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}(2\theta_B + \theta_A - 3\Delta/L)$$

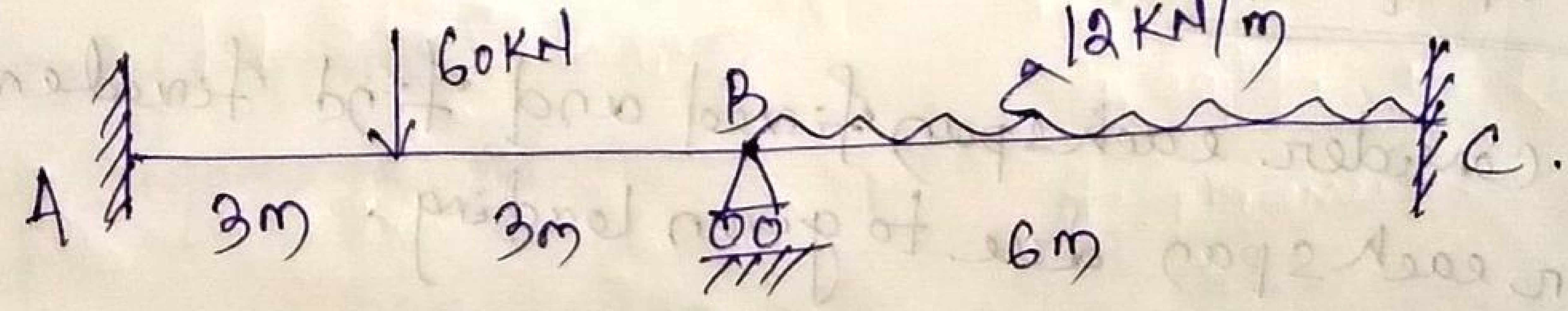
$$M_{BC} = M_{FBC} + 2EI/L\,(2\theta_B + \theta_C - 3\Delta/L)$$

$$M_{CB} = M_{FCB} + 2EI/L\,(2\theta_C + \theta_B - 3\Delta/L)$$

$$M_{CD} = M_{FCD} + 2EI/L\,(2\theta_C + \theta_D - 3\Delta/L)$$

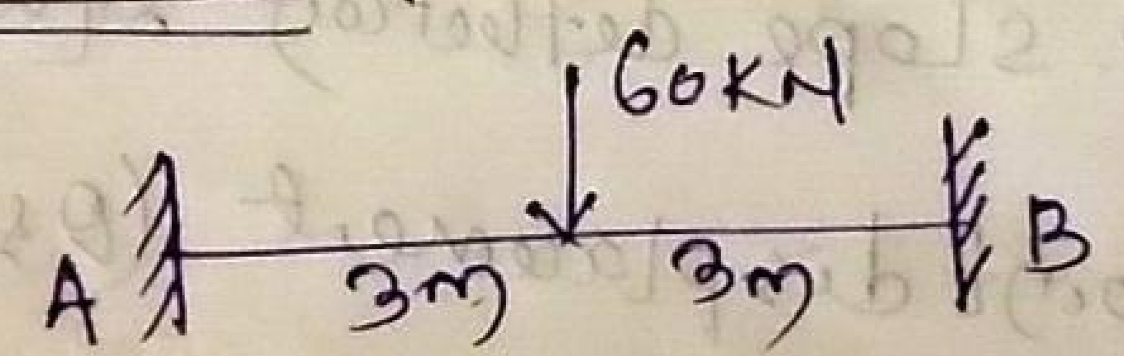
$$M_{DC} = M_{FDC} + 2EI/L\,(2\theta_D + \theta_C - 3\Delta/L)$$

Q



Step-1:- Fixed end moment

Span AB

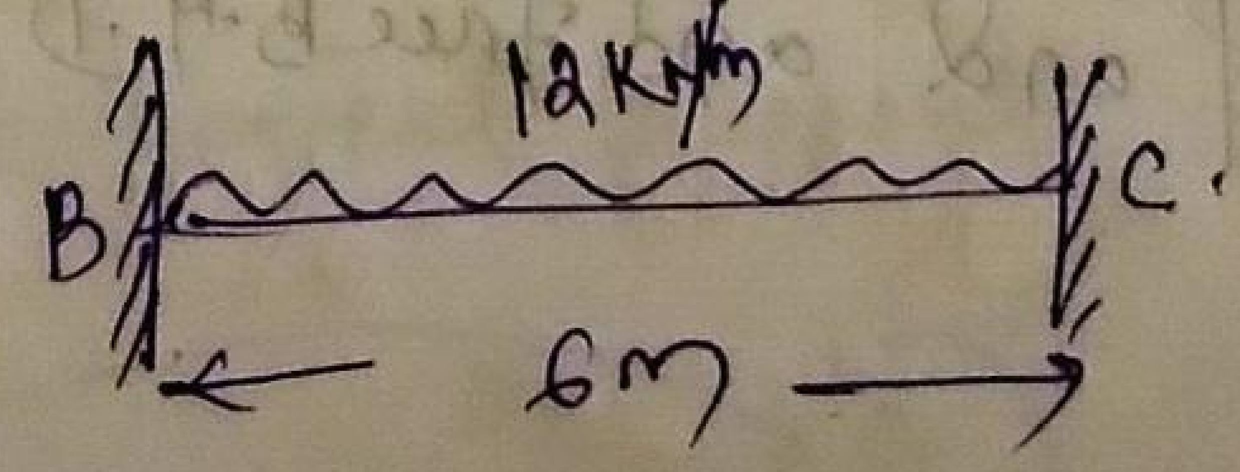


$$M_{FAB} = -wl/8$$

$$= \frac{-60 \times 6}{8} = -45 \text{ KNm}$$

$$M_{FBA} = +wl/8 = \frac{60 \times 6}{8} = 45 \text{ KNm}.$$

Span BC :-



$$M_{FBC} = \frac{-wL^2}{12} = \frac{-12\times 6^2}{12} = -36\,KNm$$

$$M_{FCB} = \frac{+wL^2}{12} = \frac{12\times 6^2}{12} = +36\,KNm$$

Step-2:- Application of slope deflection equation

$$M_{AB} = M_{FAB} + 2EI/L\,(2\theta_A + \theta_B - 3\Delta/L)$$

$$= -45 + \frac{2EI}{L}\left(\cancel{2\theta_A}^{0} + \theta_B - \cancel{3\Delta/L}^{0}\right)$$ [Here $\theta_A = 0$, $\Delta = 0$]

$$\boxed{M_{AB} = -45 + \frac{2EI}{6}\theta_B} \quad \text{(1)}$$

$$M_{BA} = M_{FBA} + 2EI/L\,(2\theta_B + \cancel{\theta_A}^{0} - 3\Delta/L)$$

$$= 45 + 2EI/6\,(2\theta_B)$$

$$\boxed{M_{BA} = 45 + 4EI/6\,\theta_B} \quad \text{(2)}$$

Member BC :-

$$M_{BC} = M_{FBC} + 2EI/L\,(2\theta_B + \cancel{\theta_C}^{0} - 3\cancel{\Delta}^{0}/L)$$

$$= -36 + 2EI/6\,(2\theta_B)$$

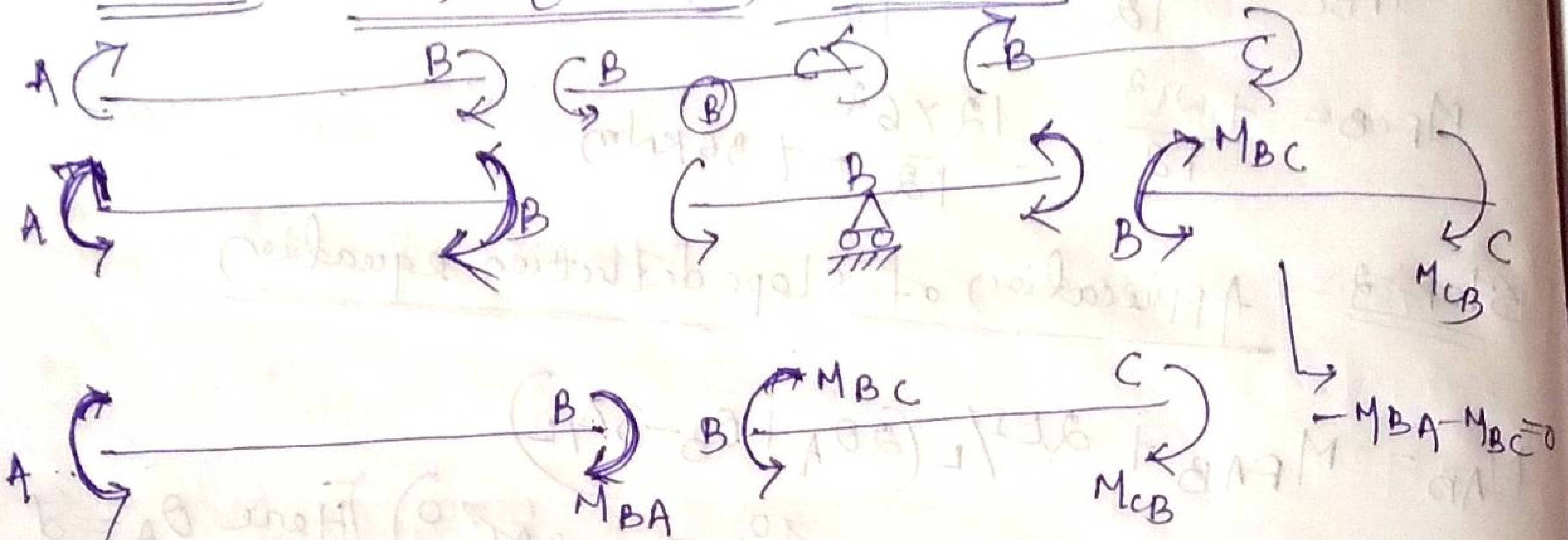
$$\boxed{M_{BC} = -36 + 4EI/6\,\theta_B} \quad \text{(3)}$$

$$M_{CB} = M_{FCB} + 2EI/L\,(\cancel{2\theta_C}^{0} + \theta_B - 3\cancel{\Delta}^{0}/L)$$

$$= 36 + 2EI/L\,(\theta_B)$$

$$\boxed{M_{CB} = 36 + 2EI/6\,\theta_B} \quad \text{(4)}$$

Step-3:- Joint equilibrium conditions:-



$M_{BA} + M_{BC} = 0$

$\Rightarrow \left[45 + \frac{4EI\theta_B}{6}\right] + \left[-36 + \frac{4EI\theta_B}{6}\right] = 0.$

$\Rightarrow \quad 9 + \frac{8EI\theta_B}{6} = 0$

$\Rightarrow \quad \theta_B = \frac{-9\times6}{8EI} = -\frac{6.75}{EI}$

Here we have unknown θ_B only. i.e rotational displacement.

Step-4:- Substituting value of θ_B in slope deflection equation we get

$M_{AB} = M_{FAB} + 2EI/L \times \theta_B$

$= -45 + 2EI/6 \times \left(-\frac{6.75}{EI}\right) = -47.25\,kNm.$

$M_{BA} = 45 + 4EI/L \times \theta_B\ {-6.75/EI}$

$= 40.5\,kNm.$

$M_{BC} = -36 + 4EI/6 \times -6.75/EI = -40.5\,kNm.$

$M_{CB} = +36 \times 2EI/6 \times -6.75/EI = +33.75\,kNm.$

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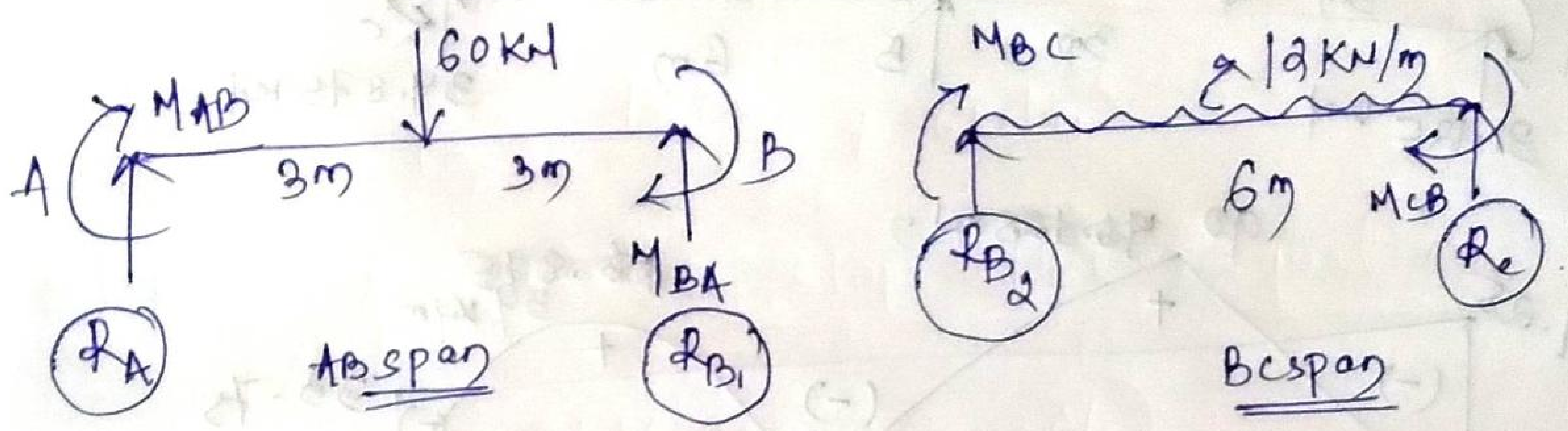
Step-5 :- Support Reaction :



$\Sigma M_B = 0$ (for span AB)

$\Rightarrow R_A \times 6 + M_{AB} + M_{BA} - 60 \times 3 = 0$

$\Rightarrow R_A = \frac{1}{6}\left[180 - (M_{AB} + M_{BA})\right]$

$= \frac{1}{6}\left[180 - (-47.25 + 40.5)\right] = 31.125$ KN ($\uparrow$)

$\Sigma M_B = 0$ (for span BC)

$\Rightarrow -R_C \times 6 + M_{BC} + M_{CB} + 12 \times 6 \times 3 = 0$

$\Rightarrow R_C = 1/6\left[216 + M_{BC} + M_{CB}\right]$

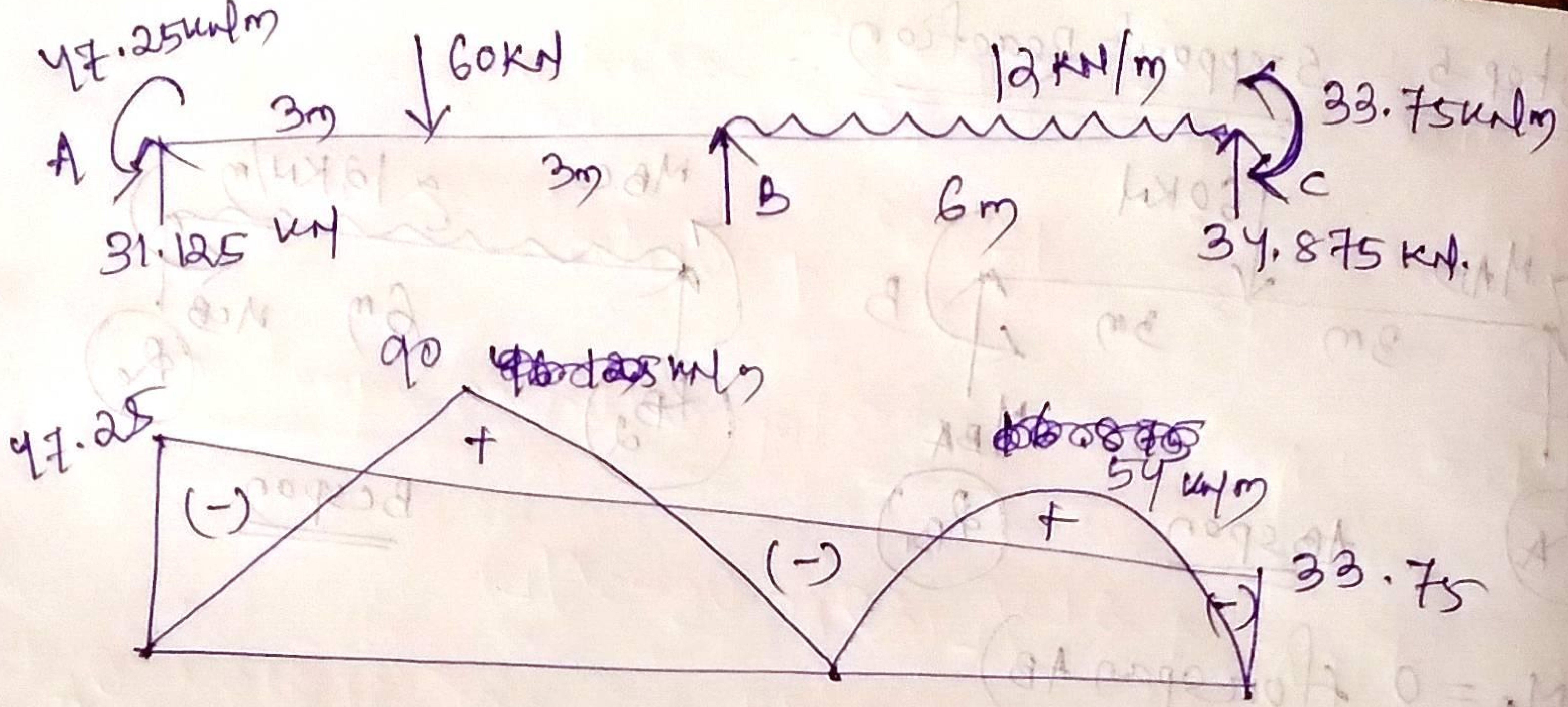
$= 34.875$ KN ($\uparrow$)

Also $\Sigma F_y = 0$ (for entire beam)

$R_A + R_B + R_C = 60 + 12 \times 6 = 132$ KN.

$\Rightarrow R_B = 132 - R_A - R_C = 132 - 31.125 - 34.875 = 66$ KN ($\uparrow$).

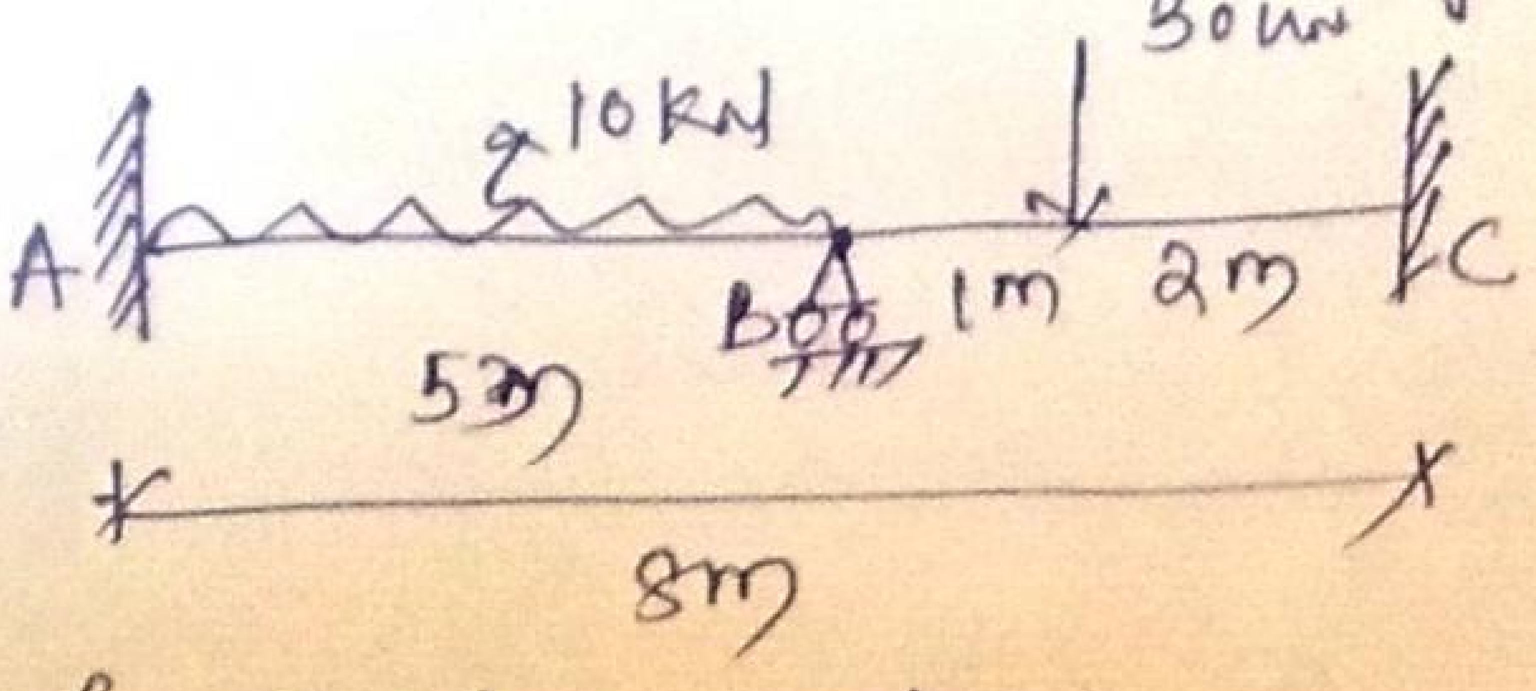
Step-6 :- Final Bending Moment diagram.



Maximum B.M for AB span $= \frac{WL}{4} = \frac{\cancel{60}^{30} \times \cancel{6}3}{\cancel{4}\cancel{2}} = 90$

" " " BC " $= \frac{WL^2}{8} = \frac{12 \times 6^2}{8} = \frac{\cancel{12}^{3} \times \cancel{36}^{18}}{\cancel{8}\cancel{4}} = 54$ KN/m

Q Draw SFD and BMD By slope deflection method. ①



(1) fixed end moment

$M_{FAB} = -wL^2/12 = -20.833\ kNm$

$M_{FBA} = +wL^2/12 = 10 \times 5^2/12 = 20.833\ kNm.$

$M_{FBC} = -wab^2/L^2 = -\dfrac{30 \times 1 \times 2^2}{3^2} = -13.33\ kNm$

$M_{FCB} = +wa^2b/L^2 = \dfrac{30 \times 1^2 \times 2}{3^2} = 6.667\ kNm.$

Step-2

-Apply slope deflection equation for each span

$M_{AB} = M_{FAB} + 2EI/L\,(\cancel{2\theta_A} + \theta_B - \cancel{3\Delta/L})$

$= -20.833 + 2EI/5\,(\theta_B) = \boxed{-20.833 + \frac{2}{5}EI\theta_B}$ —— ①

$M_{BA} = M_{FBA} + 2EI/L\,(2\theta_B + \cancel{\theta_A} - \cancel{3\Delta/L})$

$\boxed{= 20.833 + \frac{4EI}{5}\theta_B}$. —— ②

$M_{BC} = M_{FBC} + 2EI/L\,(2\theta_B + \cancel{\theta_C} - \cancel{3\Delta/L}).$

$= -13.33 + 2EI/3\,(2\theta_B) = \boxed{-13.33 + \frac{4}{3}EI\theta_B}$

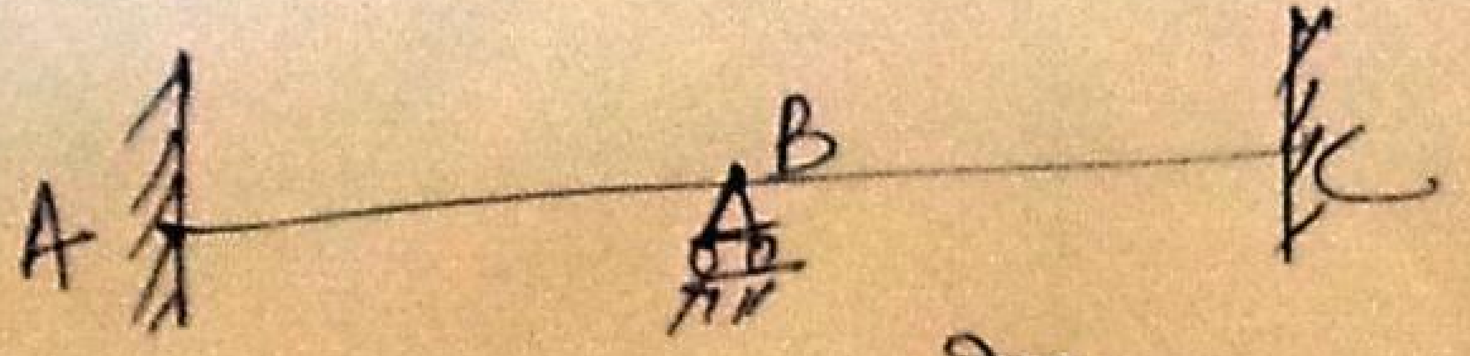
—— ③

$M_{CB} = M_{FCB} + 2EI/L\,(2\theta_C + \theta_B - 3\Delta/L).$

$$\Rightarrow \boxed{6.667 + 2EI/3\,(\theta_B)}$$

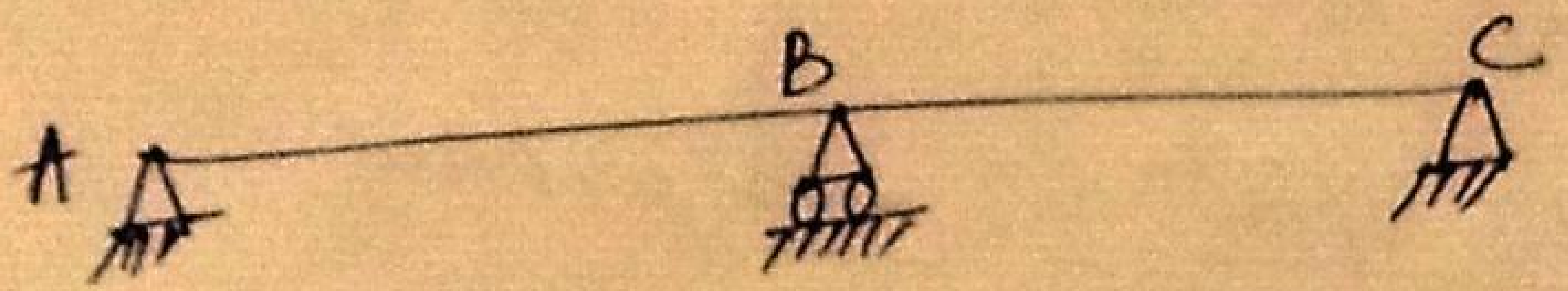
$\Rightarrow$ Step :- 3.

Apply condition of equilibrium



(1)

$M_{BA} + M_{BC} = 0 \quad [\because \Sigma F_x = 0]$



$M_{AB} = 0$ or $M_{CB} = 0$.

Apply condition of equilibrium

$M_{BA} + M_{BC} = 0.$

$$\Rightarrow \left[20.833 + \frac{4EI}{5}\theta_B\right] + \left[-13.33 + 4/3\, EI\,\theta_B\right] = 0.$$

$\Rightarrow EI\theta_B = -3.51.$

Step-4 :- putting value of $EI\theta_B$. in eq^n(1) to (4)

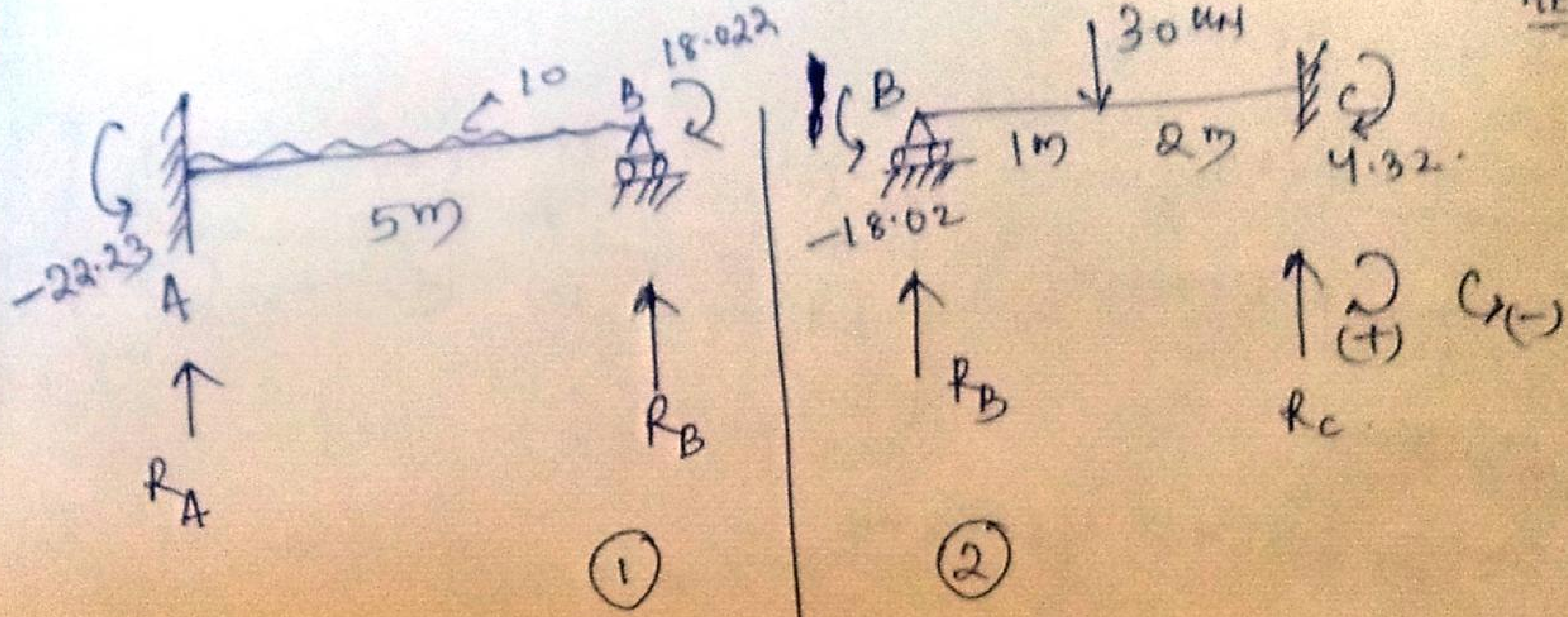
$M_{AB}^{\checkmark} = -20.833 + 2/5(-3.51) = -22.23$ KNm

$M_{BA}^{\checkmark} = 20.833 + 4/5\,(-3.51) = 18.022$ KNm

$M_{BC}^{\checkmark} = -13.33 + 4/3\,(-3.51) = -18.01$ KNm

$M_{CB}^{\checkmark} = 6.667 + \frac{2}{3}(-3.51) = 4.32$ KNm

Final Moment and find support Reaction by separating AB and BC (2)



$\circlearrowright$ (+) $\circlearrowleft$ (-)

$\Sigma M_A = 0$

$-22.23 + 10 \times 5 \times \frac{5}{2} + 18.022 - R_B \times 5 = 0$

$\Rightarrow \boxed{R_B = 24.15 \text{ kN}}$

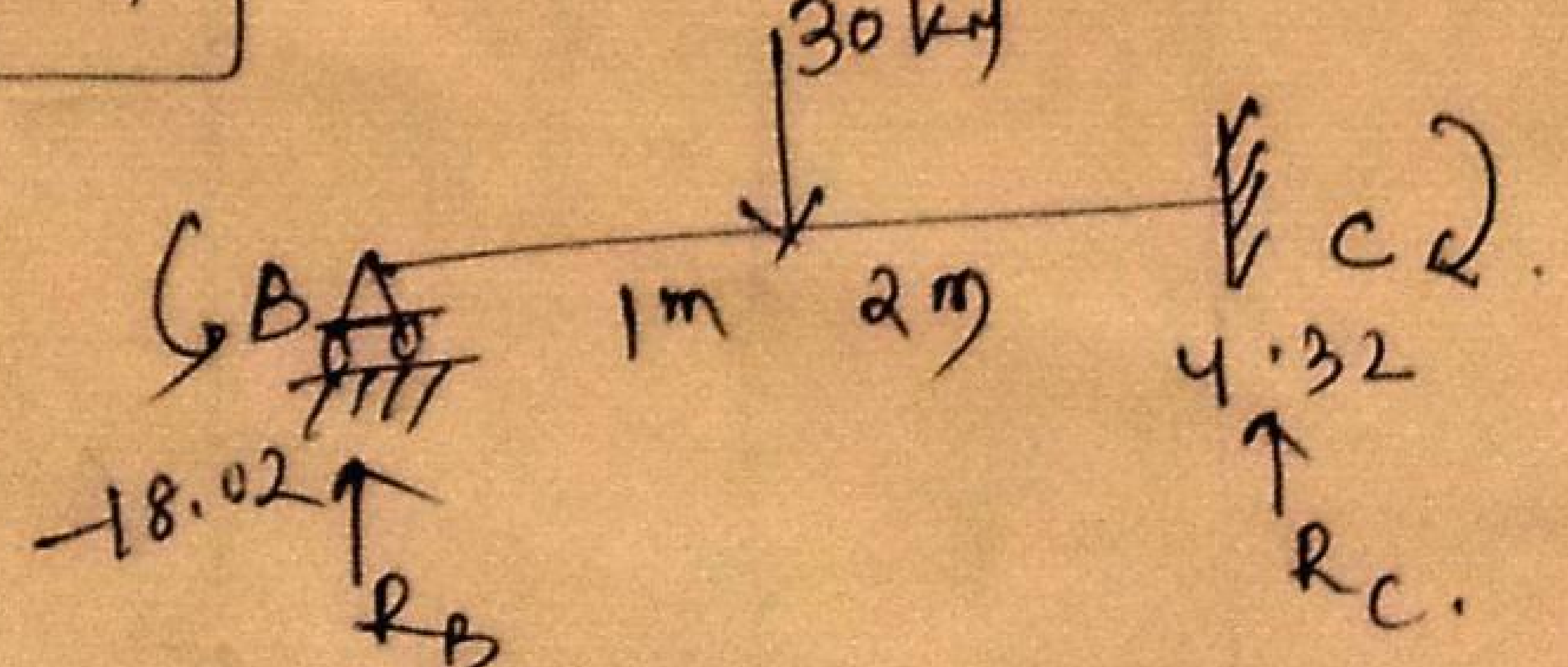
when we convert udl to point load that will deflect downward downward means -ve.

$\Sigma F_y = 0$. $(\uparrow)(+)$ $\downarrow(-)$

$R_A + 24.15 - 10 \times 5 = 0$.

$\Rightarrow \boxed{R_A = 25.85 \text{ kN}}$

for fig-2



$\Sigma M_B = 0$.

$\Rightarrow -18.01 + 30 \times 1 - R_C \times 3 + 4.32 = 0$.

$\Rightarrow \boxed{R_C = 5.43 \text{ kN}}$

$\Sigma F_y = 0.$

$R_B + 5.43 - 30 = 0.$

$\Rightarrow \boxed{R_B = 24.57 \text{ kN}}$

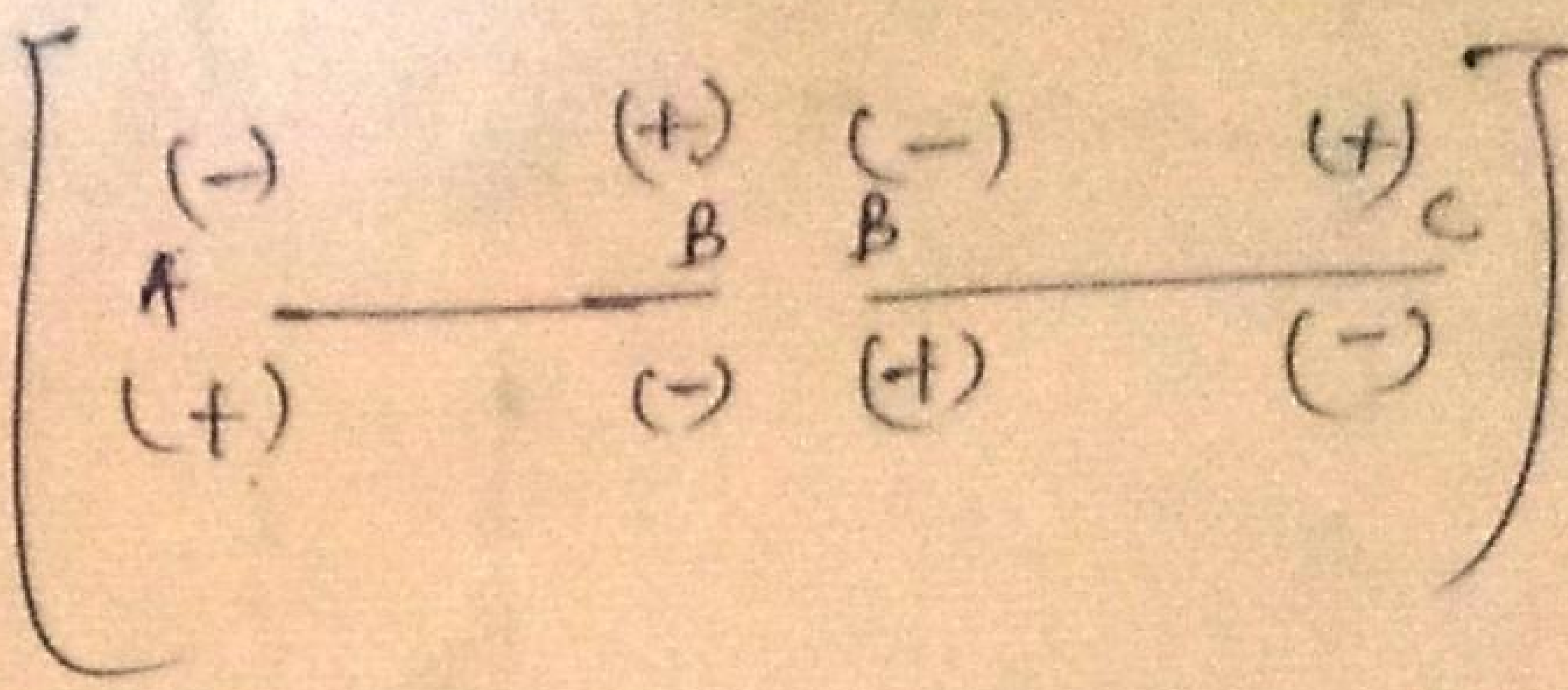
6) Final support Reaction.

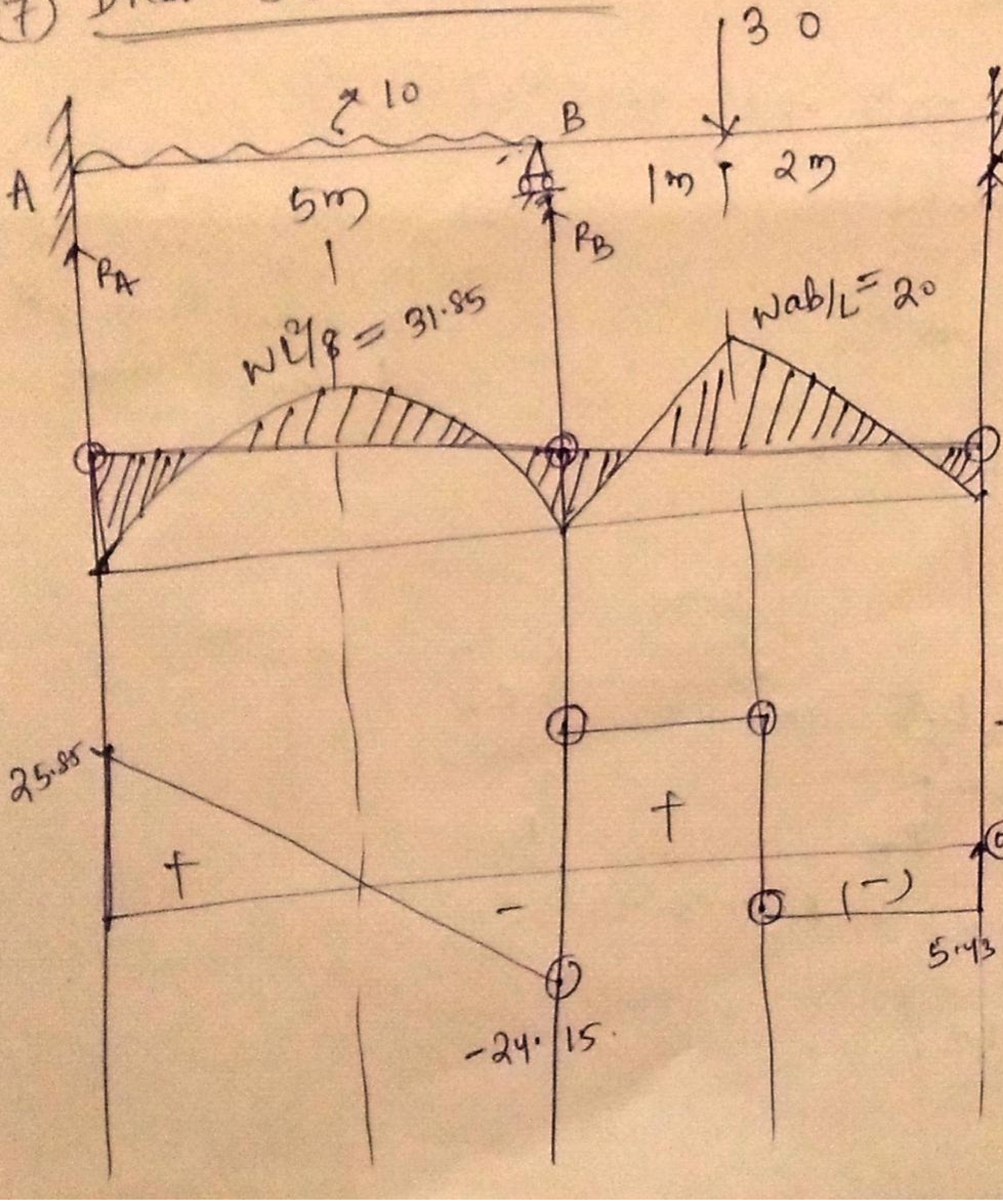
$R_A = 25.85 \text{ kN}.$

$R_B = 24.15 + 24.57 = 48.72 \text{ kN}.$

$R_C = 5.43 \text{ kN}.$

(7) Draw SFD and BMD





⇒ Net BMD

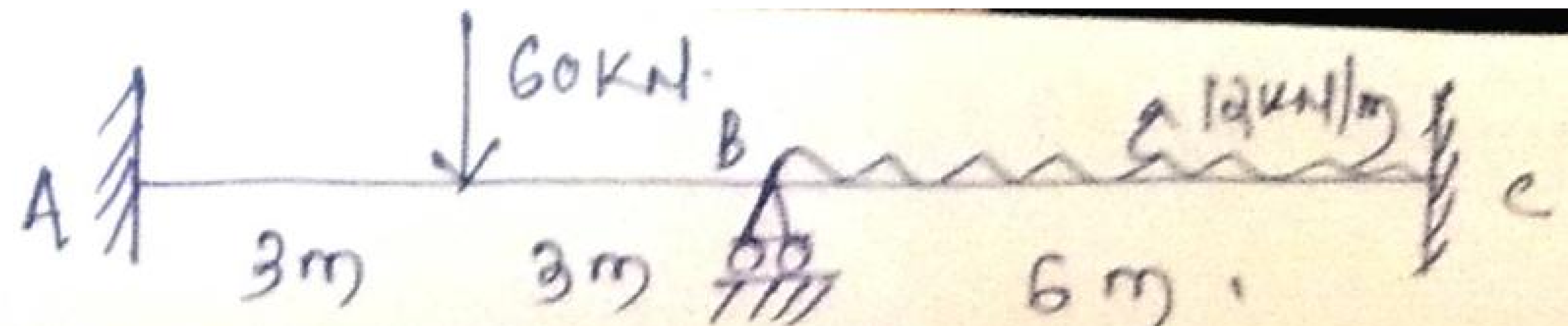
−ve should be above NA

A → 25.85 − 10×5
= −24.15 kN

B → −24.15 + 48.72
= 24.57

C → 24.57 − 30
= −5.43 + 5.43 = 0

(3) ③



Solution

Step-1 fixed end moment

$$M_{FAB} = -WL/8 = \frac{-60\times 6}{8} = -45\,\text{KNm}$$

$$M_{FBA} = +WL/8 = +45\,\text{KNm}$$

$$M_{FBC} = -wL^2/12 = -12\times 6^2/12 = -36\,\text{KNm}$$

$$M_{FCB} = +wL^2/12 = +36\,\text{KNm}$$

Step-2: Apply slope deflection equation for each span.

$$M_{AB} = M_{FAB} + 2EI/L\,(\cancel{2\theta_A}^{0} + \theta_B - \cancel{3\Delta/L}^{0})$$

$$= \boxed{-45 + 2EI/6\,(\theta_B)} \quad \text{———} \; (1)$$

$$M_{BA} = 45 + 2\theta_B\,(2EI/6) = \boxed{45 + 4EI/6\,\theta_B} \quad \text{———} \; (2)$$

$$M_{BC} = \boxed{-36 + 4EI/6\,\theta_B} \quad \text{———} \; (3)$$

$$M_{CB} = \boxed{36 + 2EI/6\,\theta_B} \quad \text{———} \; (4)$$

Step-3: Apply condition of equilibrium:

$$M_{BA} + M_{BC} = 0.$$

$$\Rightarrow \left[45 + 4EI/6\,\theta_B\right] + \left[-36 + 4EI/6\,\theta_B\right] = 0.$$

$$\boxed{EI\theta_B = -6.75}$$

Step-4 :- putting value of θ_B in eq.ⁿ (1), (2) (3) & (4)

$M_{AB} = -47.25$ KNm

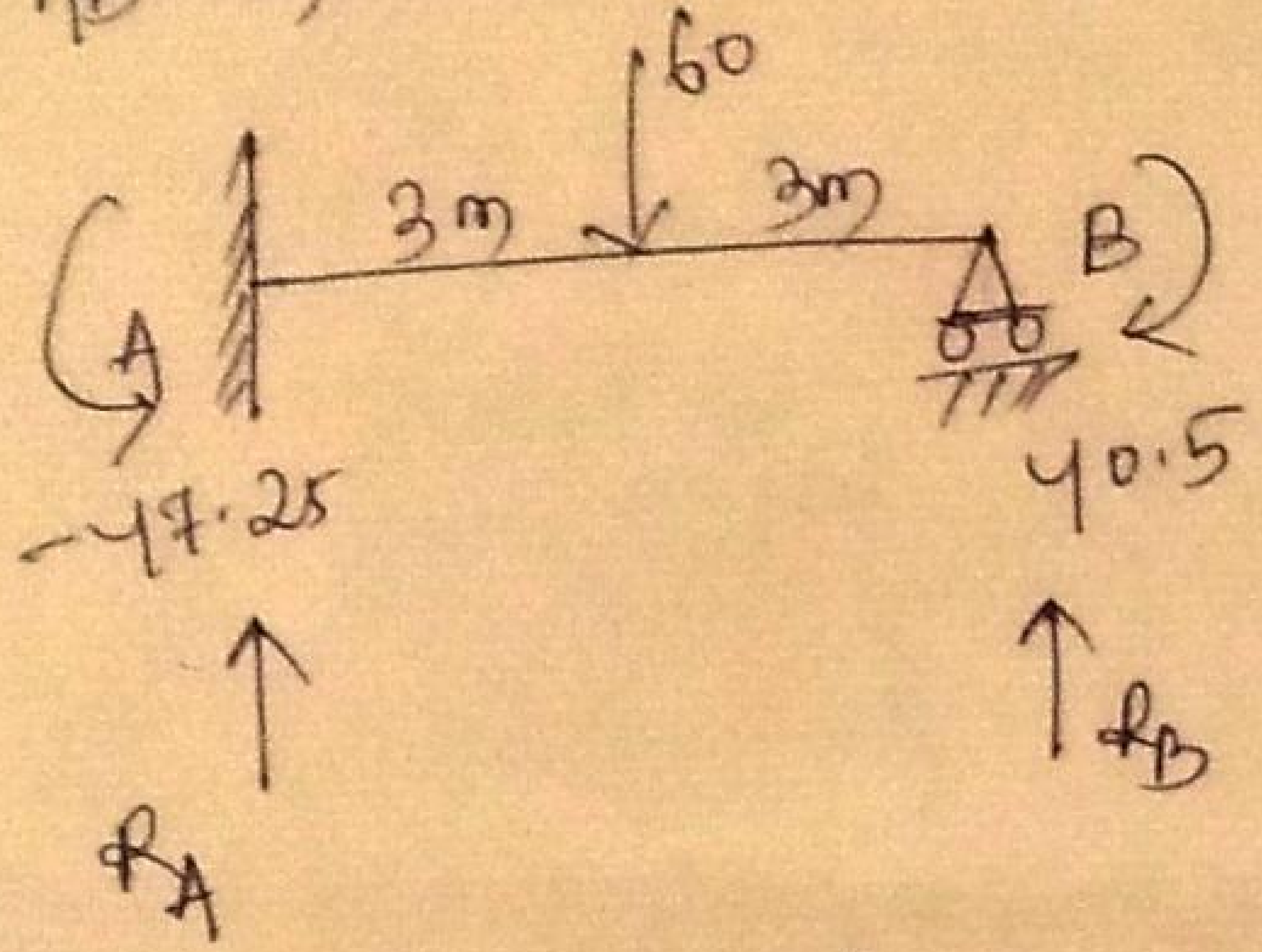
$M_{BA} = 40.5$ KNm

$M_{BC} = -40.5$ KNm

$M_{CB} = +33.75$ KNm.

[anticlock = -ve
clock = +ve] for support reaction.

Step-5 :- Final moment and find support Reaction by separating AB and BC.



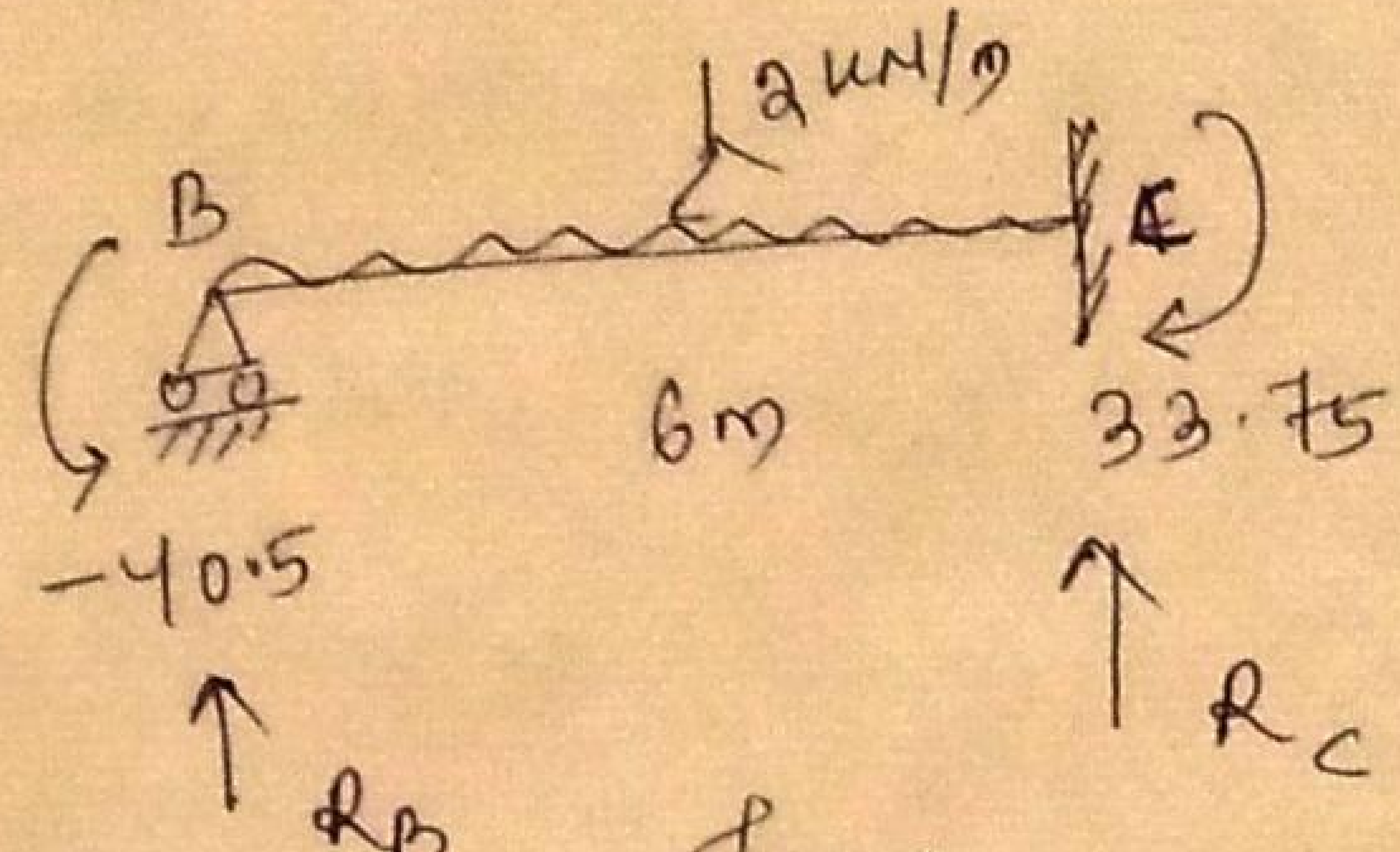


fig-2

$\Sigma M_A = 0$ (for fig-1)

$\Rightarrow -47.25 + 60 \times 3 + 40.5 - R_B \times 6 = 0$

$\Rightarrow \boxed{R_B = 28.875 \text{ kN}} \uparrow$.

$\Sigma F_y = 0$. or (upward force = downward force)

$R_A + 28.875 = 60$

$\Rightarrow \boxed{R_A = 31.125 \text{ KN}} \uparrow$.

for figure-2 $\quad \Sigma M_B = 0$.

$\Rightarrow -40.5 + 12 \times 6 \times 6/2 + 33.75 - R_C \times 6 = 0$

$\Rightarrow \boxed{R_C = 34.875} \text{ kN}$

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If we take $\Sigma f_y = 0$ or upward force = downward force (4)

$\Rightarrow R_B + 34.875 = 12 \times 6$

$\Rightarrow \boxed{R_B = 37.125 \text{ kN}}$

(6) Final support Reaction

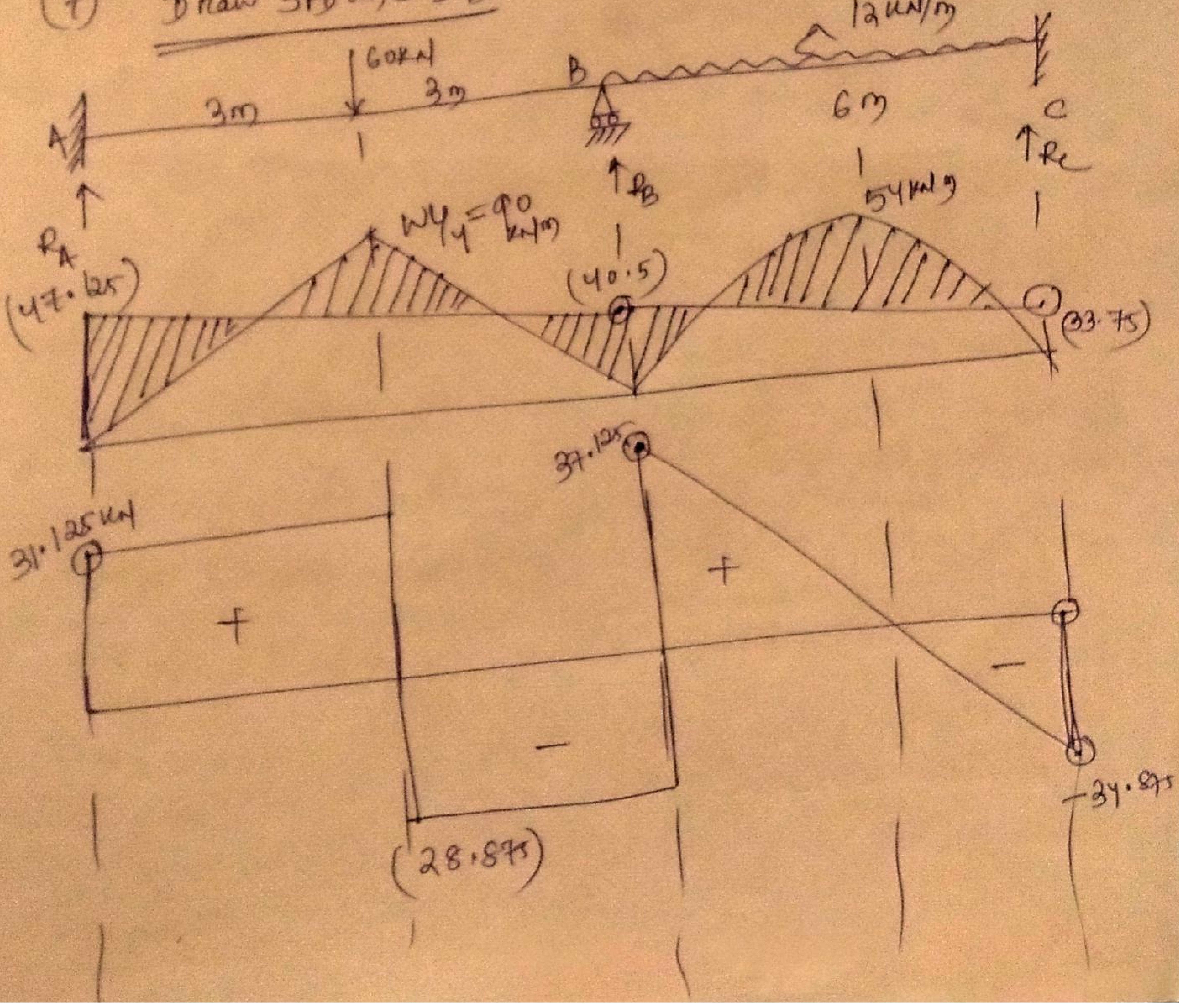
(−)	(+)	(−)	(+)
A (+)	(−) B	B (+)	(−) C

$R_A = 31.125 \text{ kN}$

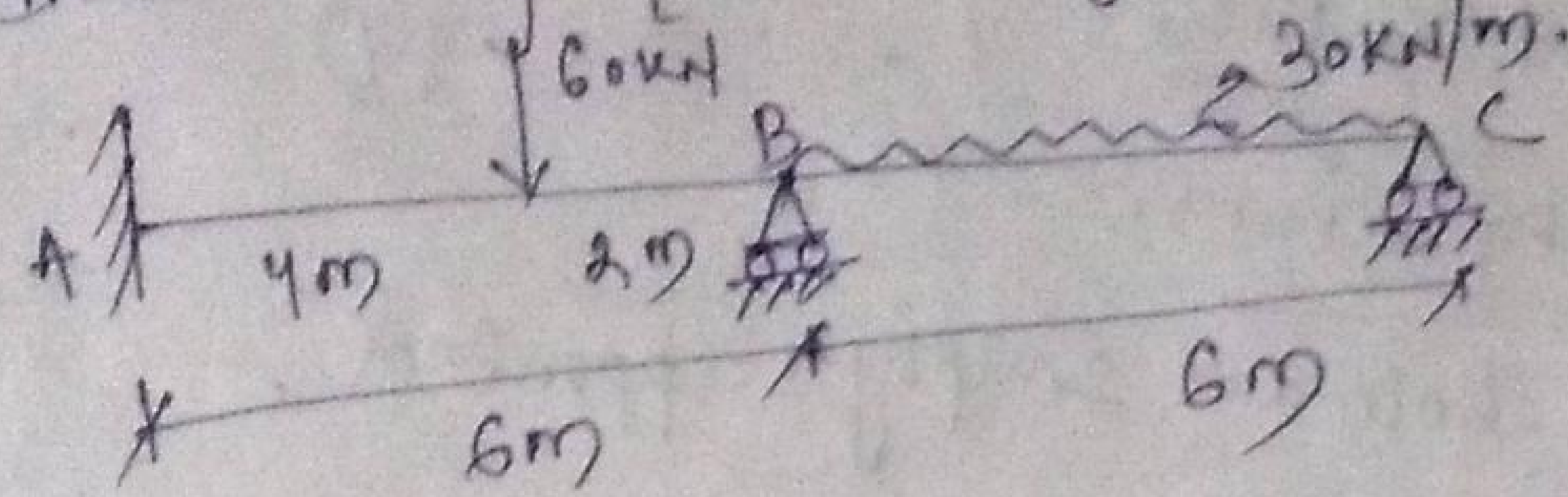
$R_B = 37.125 + 28.875 = 66 \text{ kN}$

$R_C = 34.875 \text{ kN}$

(7) Draw SFD and BMD



(3) Analyse the continuous beam shown in figure and draw Bending Moment diagram.



Step-1:- fixed end Moment

$M_{FAB} = -wab^2/L^2 = -60\times4\times2^2/6^2 = -26.67 kNm$

$M_{FBA} = +wa^2b/L^2 = \dfrac{+60\times4^2\times2}{6^2} = +53.33 kNm$

$M_{FBC} = \dfrac{-wL^2}{12} = -30\times6^2/12 = -90 kNm$.

$M_{FCB} = +wL^2/12 = +90 kNm$.

Step-2: Deflection equations

$M_{AB} = M_{FAB} + 2EI/L\,(\cancel{2\theta_A}^{0} + \theta_B - \cancel{3\Delta/L}^{0})$

$= -26.67 + 2EI/6\,(\theta_B)$

$= -26.67 + 1/3\,EI\theta_B$ ——— (1)

$M_{BA} = 53.33 + 2EI/6\,(2\theta_B + \cancel{\theta_A}^{0} - \cancel{3\Delta/L}^{0})$

$= 53.33 + 2/3\,EI\theta_B$ ——— (2)

$M_{BC} = -90 + 2EI/6\,(2\theta_B + \theta_C - \cancel{3\Delta/L}^{0})$

$= -90 + 2/3\,EI\theta_B + 1/3\,EI\theta_C$ ——— (3)

$M_{CB} = 90 + 2EI/6 (\theta_B + 2\theta_C - \cancel{3\Delta/L})^{0}$

$= \boxed{90 + 1/3\, EI\theta_B + 2/3\, EI\theta_C}$ ——— (4).

Step:-3 ;- Equilibrium equations:-

As B and C both are simply supported.

$\Sigma M_B = 0$

$M_{BA} + M_{BC} = 0$

$\Rightarrow 53.33 + (2/3\, EI\theta_B) - (90 + 2/3\, EI\theta_B + 1/3\, EI\theta_C) = 0$.

$\Rightarrow \boxed{4EI\theta_B + EI\theta_C = 110}$ ——— (5)

$\Sigma M_C = 0$. Means only $M_{CB} = 0$.

$\Rightarrow \boxed{90 + 1/3\, EI\theta_B + 2/3\, EI\theta_C = 0}$

$\Rightarrow \boxed{EI\theta_B + 2EI\theta_C = -270}$ ——— (6).

Solving equation (5) and (6) we will get

$EI\theta_B = 70$

$EI\theta_C = -170$.

Step-4:- Final Moment Putting value of $EI\theta_B$ and $EI\theta_C$ in eq.^{n} (1) to (4)

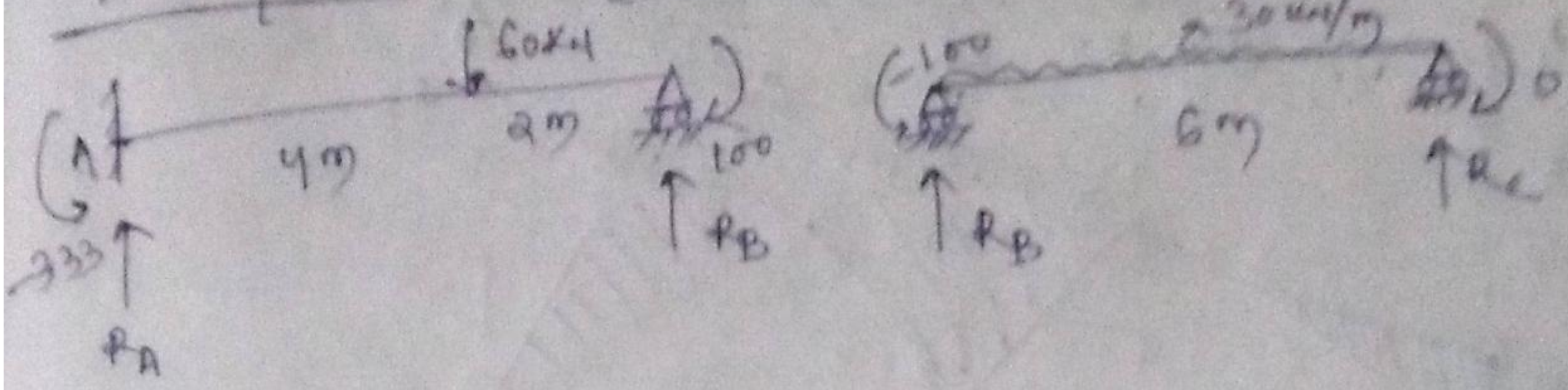
$M_{AB}' = -3.33$ KNm

$M_{BA}' = 100$ KNm

$M_{BC}' = -100$ KNm

$M_{CB}' = 0$.

Final Moment and find support Reaction by separating AB & BC



$\Sigma M_A = 0$

$\Rightarrow -3.33 + 60\times4 + 100 - R_B\times6 = 0$

$\Rightarrow \boxed{R_B = 56.11\ kN}$

$\Sigma F_y = 0 \cdot (\uparrow = \downarrow)$

$\Rightarrow R_A + R_B = 60$

$\Rightarrow R_A + 56.11 = 60$

$\Rightarrow \boxed{R_A = 3.89\ kN}$

$\Sigma M_B = 0$

$\Rightarrow -100 + 30\times6\times3 - R_C\times6 = 0$

$\Rightarrow \boxed{R_C = 73.33\ kN}$

$\Sigma F_y = 0\ (\uparrow = \downarrow)$

$\Rightarrow R_B + R_C = 30\times6$

$\Rightarrow R_B + 73.33 = 180$

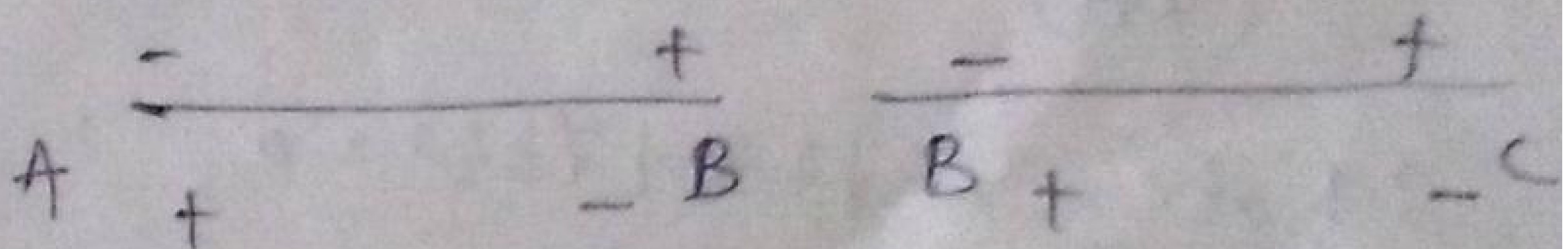
$\Rightarrow \boxed{R_B = 106.67\ kN}$

Support Reaction

$R_A = 3.89\ kN$

$R_B = 162.78\ kN$

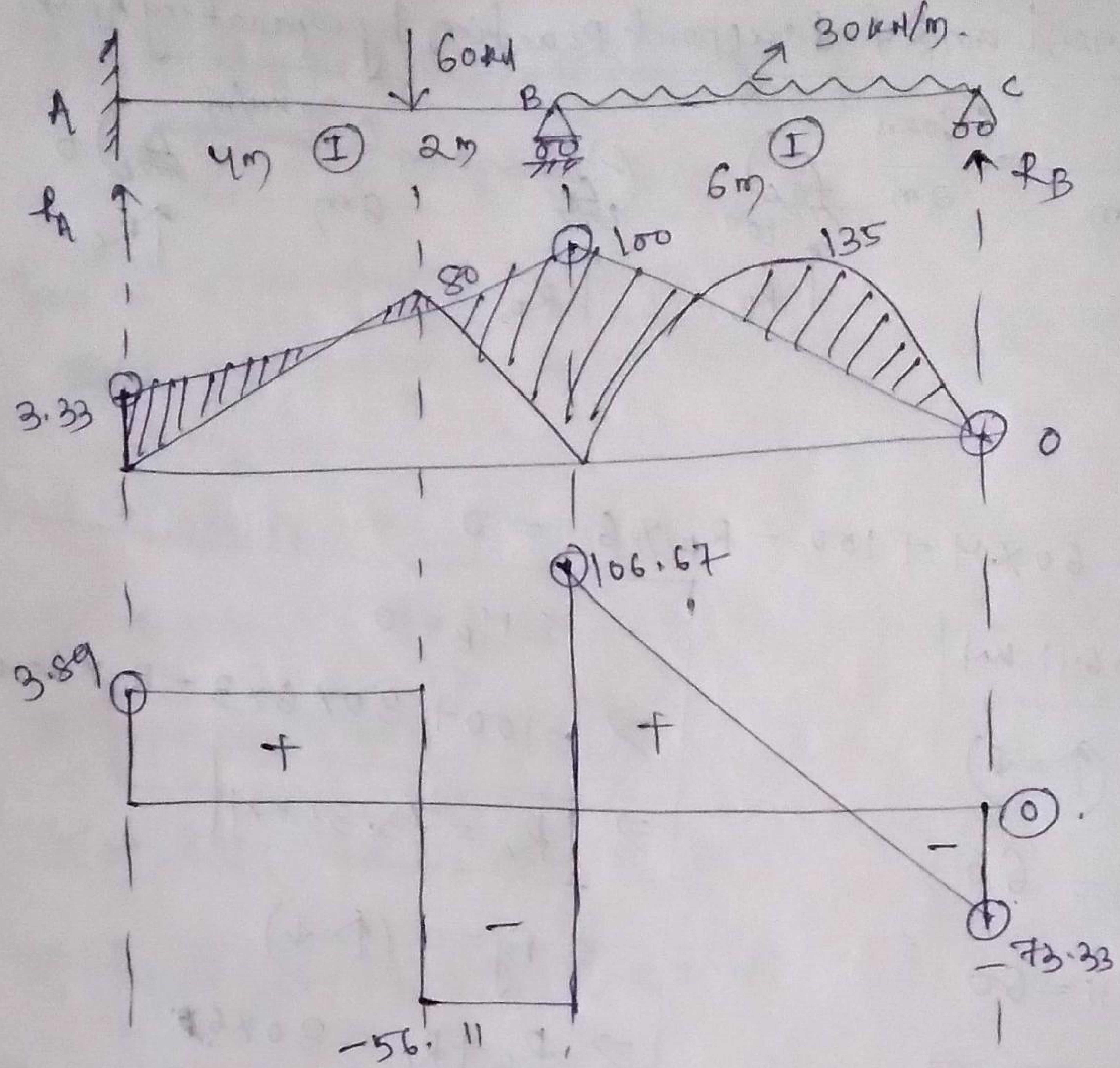
$R_C = 73.33\ kN$



Step-7: Draw SFD and BMD

Free B.M for AB $= Wab/L = \frac{60\times4\times2}{6} = 80\ kNm$

for BC $= wl^2/8 = 30\times6^2/8 = 135\ kNm$



S.F at A = $+R_A$ = 3.89 kN

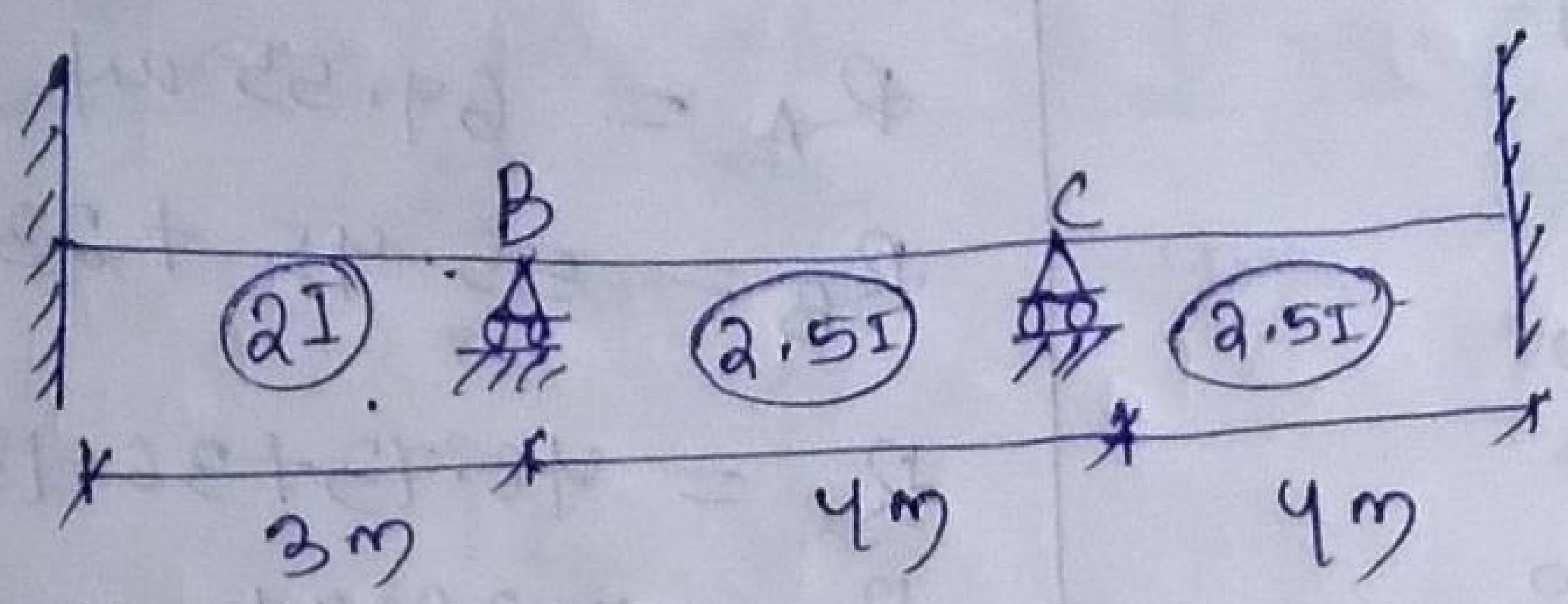
S.F at left of B = 3.89 − 60 = −56.11 kN.

S.F at B = −56.11 + 162.78 = 106.67 kN.

S.F at C (left) = 106.67 − (30 × 6) = −73.33

S.F at C = −73.33 + 73.33 = 0

Q. Analyse the continuous beam in figure by slope deflection method. Support B settles down by 5mm. Take $E = 2\times10^5 \ N/mm^2$ and $I = 36\times10^6 \ mm^4$.



As no Load so fixed end Moment can't calculated.

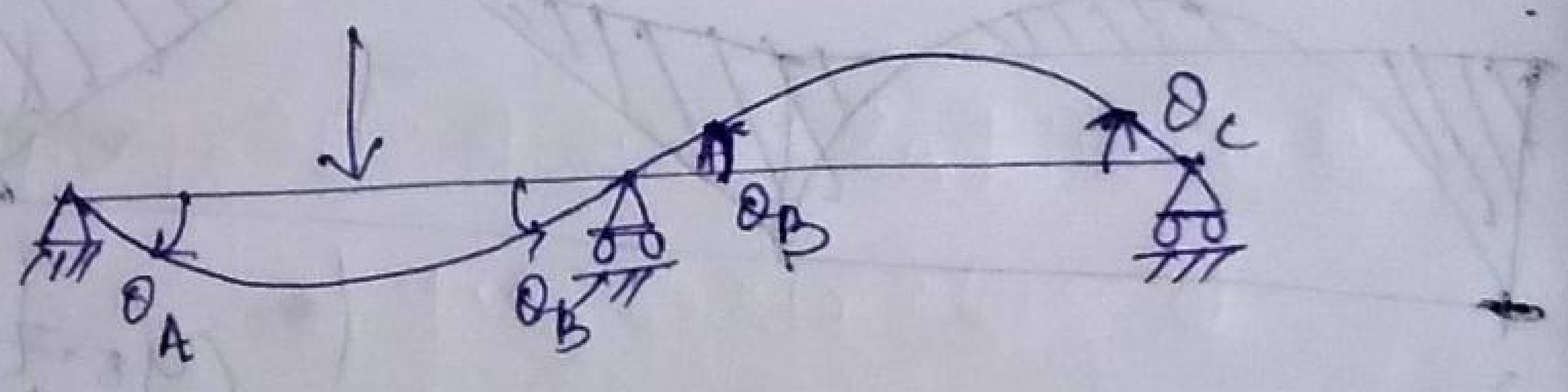
Slope deflection equation

Member AB

$$M_{AB} = M_{FAB} + \frac{2E(2I)}{L}\left(2\theta_A + \theta_B - 3\Delta/L\right)$$

Here $M_{FAB} = 0$, $\theta_A = 0$ and $\Delta = 0.005m$.

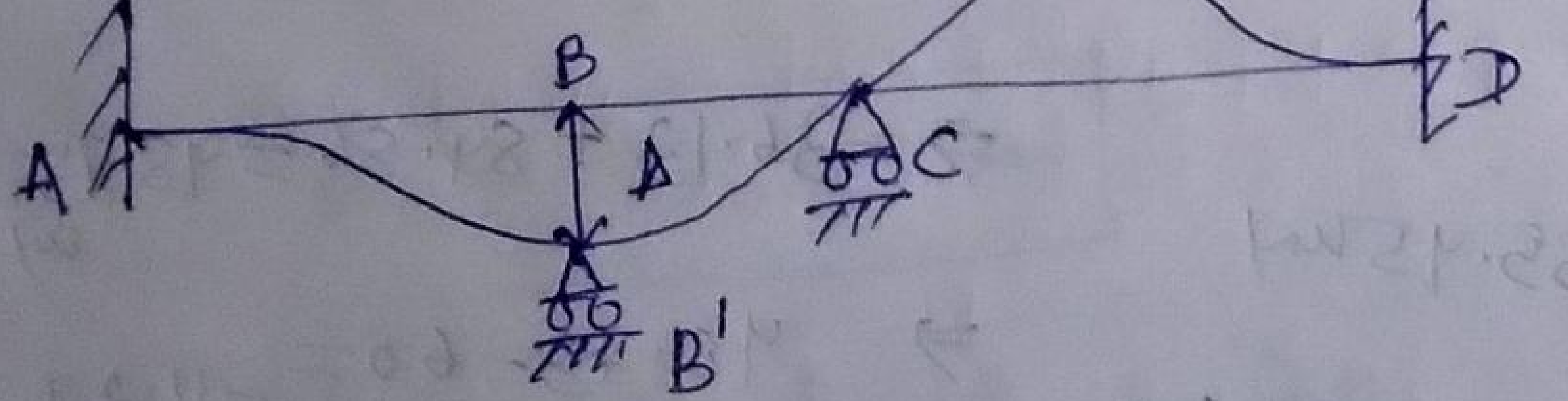
Introduce clockwise rotation.



θ_A = clockwise = +ve

θ_B = anticlockwise = -ve

θ_C = clockwise = +ve



$$M_{AB} = \frac{4EI}{3}\left(0 + \theta_B - \frac{3\times 0.005}{3}\right)$$

$$= \boxed{1.33\ EI\theta_B - 6.67 \times 10^{-3}\ EI} \quad (1)$$

$$M_{BA} = \cancel{M_{FBA}}^{0} + \frac{2E(2I)}{L}\left(2\theta_B + \theta_A - 3\Delta/L\right)$$

$$= 4EI/3\left(2\theta_B + 0 - \frac{3\times 0.005}{3}\right)$$

$$= \boxed{2.67\ EI\theta_B - 6.67\times 10^{-3}\ EI} \quad (2)$$

Member BC.

$$M_{BC} = M_{FBC} + \frac{2EI(2.5I)}{L}\left(2\theta_B + \theta_C - 3\Delta/L\right)$$

Here $M_{FBC} = 0$ and $\Delta = -0.005\,m$ (produce anticlockwise rotation)

$$M_{BC} = \cancel{M_{FBC}}^{0} + 5EI/4\left(2\theta_B + \theta_C - 3\Delta/L\right)$$

$$= 0 + 5EI/4\left(2\theta_B + \theta_C + 3\times 0.005/4\right)$$

$$= \boxed{2.5\ EI\theta_B + 1.25\ EI\theta_C + 4.6875\times 10^{-3}\ EI} \quad (3)$$

$$M_{CB} = \cancel{M_{FCB}}^{0} + \frac{2E(2.5I)}{L}\left(2\theta_C + \theta_B - 3\Delta/L\right)$$

$$= 0 + 5EI/4\left(2\theta_C + \theta_B + 3\times 0.005/4\right)$$

$$= \boxed{2.5\ EI\theta_C + 1.25\ EI\theta_B + 4.6875\times 10^{-3}\ EI} \quad (4)$$

Member CD :-

$$M_{CD} = M_{FCD} + \frac{2E(2.5I)}{L}\left(2\theta_C + \theta_D - 3\Delta/L\right)$$

Here $M_{FCD} = 0$, $\theta_D = 0$ and $\Delta = 0$.

$$M_{CD} = \cancel{M_{FCD}}^{0} + \frac{2EI(2.5)}{4}(2\theta_c)$$

$$= \boxed{2.5\,EI\theta_c} \quad (5)$$

$$M_{DC} = \cancel{M_{FDC}}^{0} + \frac{2E(2.5I)}{L}(\cancel{2\theta_D}^{0} + \theta_c - \cancel{3\Delta/L}^{0})$$

$$= \boxed{0 + 5EI/4\ \theta_c} \quad (6)$$

$$= \boxed{1.25\,EI\theta_c}$$

Equilibrium Equations.

Consider joint equilibrium of joint B

$$\boxed{M_{BA} + M_{BC} = 0}$$

$$\Rightarrow 2.67EI\theta_B - 6.67\times10^{-3}EI + 2.5EI\theta_B + 1.25EI\theta_c + 4.6875\times10^{-3}EI = 0$$

$$\Rightarrow 5.17EI\theta_B + 1.25EI\theta_c - 1.9825\times10^{-3}EI = 0. \quad (A)$$

Now consider joint 'C'

$$\boxed{M_{CB} + M_{CD} = 0}$$

$$\Rightarrow 2.5EI\theta_c - 1.25EI\theta_B + 4.6875\times10^{-3}EI + 2.5EI\theta_B = 0$$

(B)

on solving equation (A) and (B) we get

$\theta_B = 6.49\times10^{-4}$

$\theta_C = -1.099\times10^{-3}$.

Final end moments

on putting the value of θ_B and θ_C in slope defection equations we get.

$M_{AB} = -41.76$ KNm

$M_{BA} = -35.49$ KNm

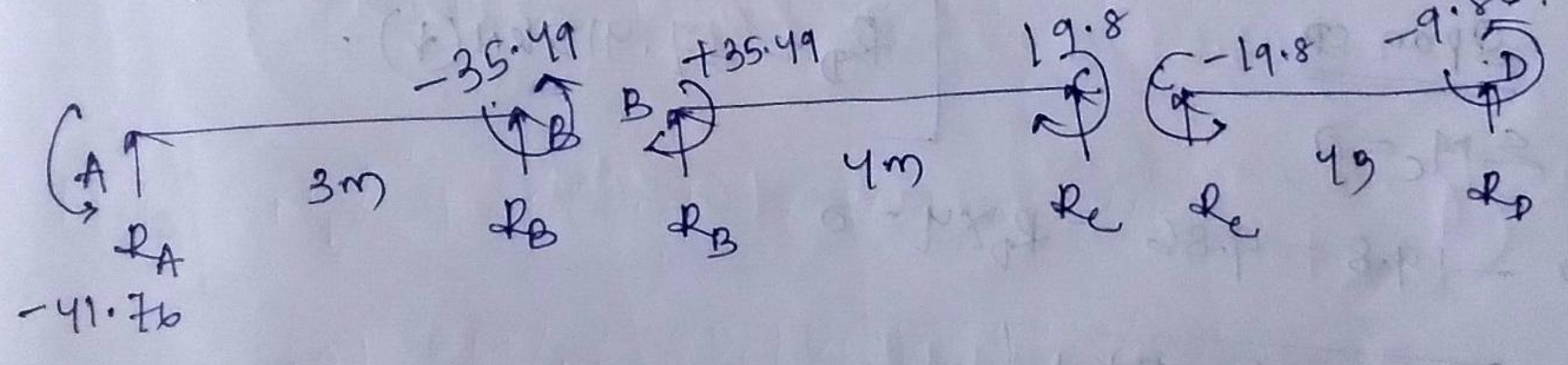
$M_{BC} = +35.49$ KNm

$M_{CB} = 19.8$ KNm

$M_{CD} = -19.8$ KNm

$M_{DC} = -9.864$ KNm

Support Reaction



Span AB

$\Sigma M_A = 0$

$\Rightarrow -41.76 - 35.49 - R_B\times3 = 0$

$\Rightarrow R_B = -25.75$ KN (↓)

$\Sigma f_y = 0$. $(\uparrow = \downarrow)$

$\Rightarrow R_A + R_B = 0$

$\Rightarrow R_A + (-25.75) = 0$

$\Rightarrow \boxed{R_A = 25.75\ KN (\uparrow)}$

Span BC :-

$\Sigma M_B = 0$

$\Rightarrow 35.49 + 19.8 - R_e \times 4 = 0$

$\Rightarrow \boxed{R_e = 13.82 (\uparrow)}$

$\uparrow = \downarrow$

$R_B + R_e = 0$

$\Rightarrow R_B + 13.82 = 0$

$\Rightarrow \boxed{R_B = -13.82 KN (\downarrow)}$

Span CD

$\Sigma M_C = 0$

$\Rightarrow -19.8 - 9.86 - R_D \times 4 = 0$

$\Rightarrow \boxed{R_D = -7.415\ KN}$

$(\uparrow = \downarrow) \Rightarrow R_C + R_D = 0$

$\Rightarrow R_e + (-7.415) = 0$

$\Rightarrow \boxed{R_e = 7.415\ KN (\uparrow)}$

Final support Reaction

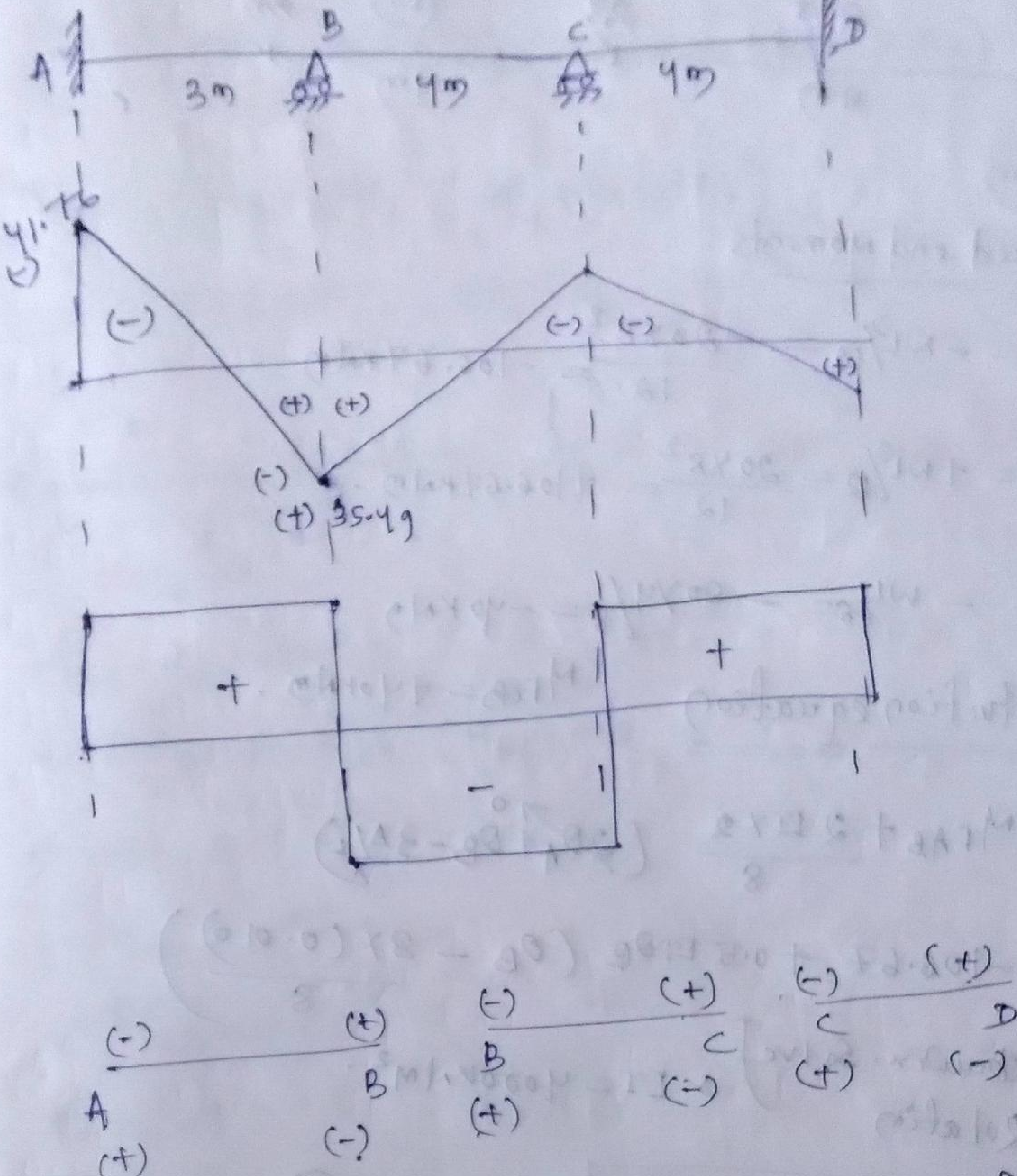
$R_A = 25.75 KN (\uparrow)$

$R_B = R_{B_1} + R_{B_2} = -39.57 (\downarrow)$

$R_e = R_{e_1} + R_{e_2} = 21.236 KN (\uparrow)$

$R_D = -7.415 KN (\downarrow)$.

SFD and BMD

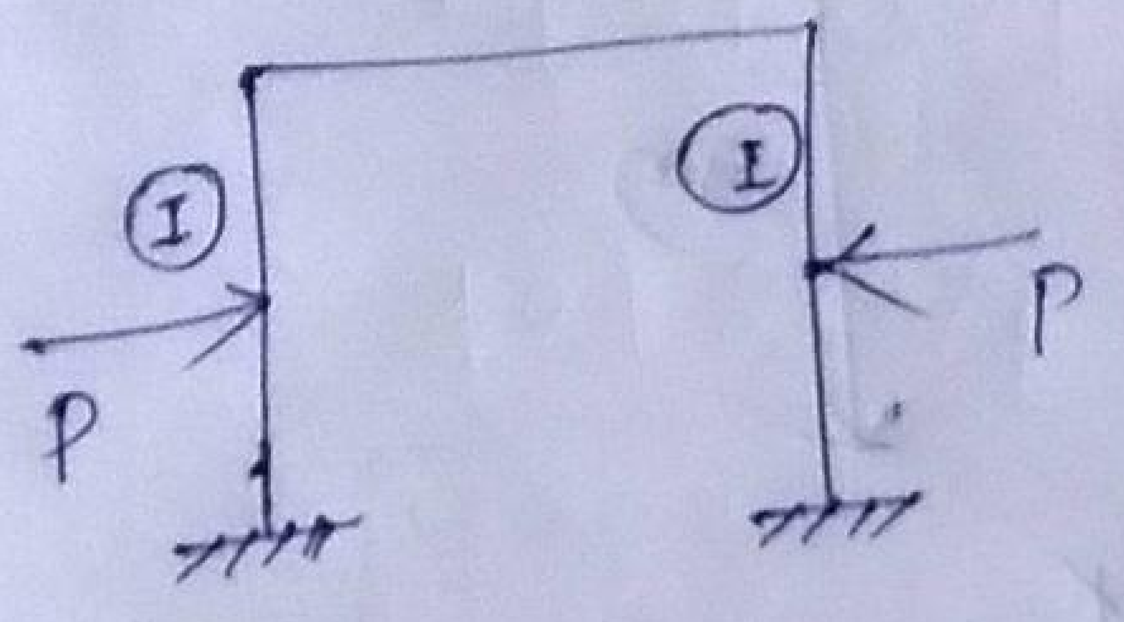


Q Analyse the continuous beam as shown in fig. by slope deflection method. if joint B sinks by 10mm given $EI = 4000 kNm^2$. Draw BMD and SFD.

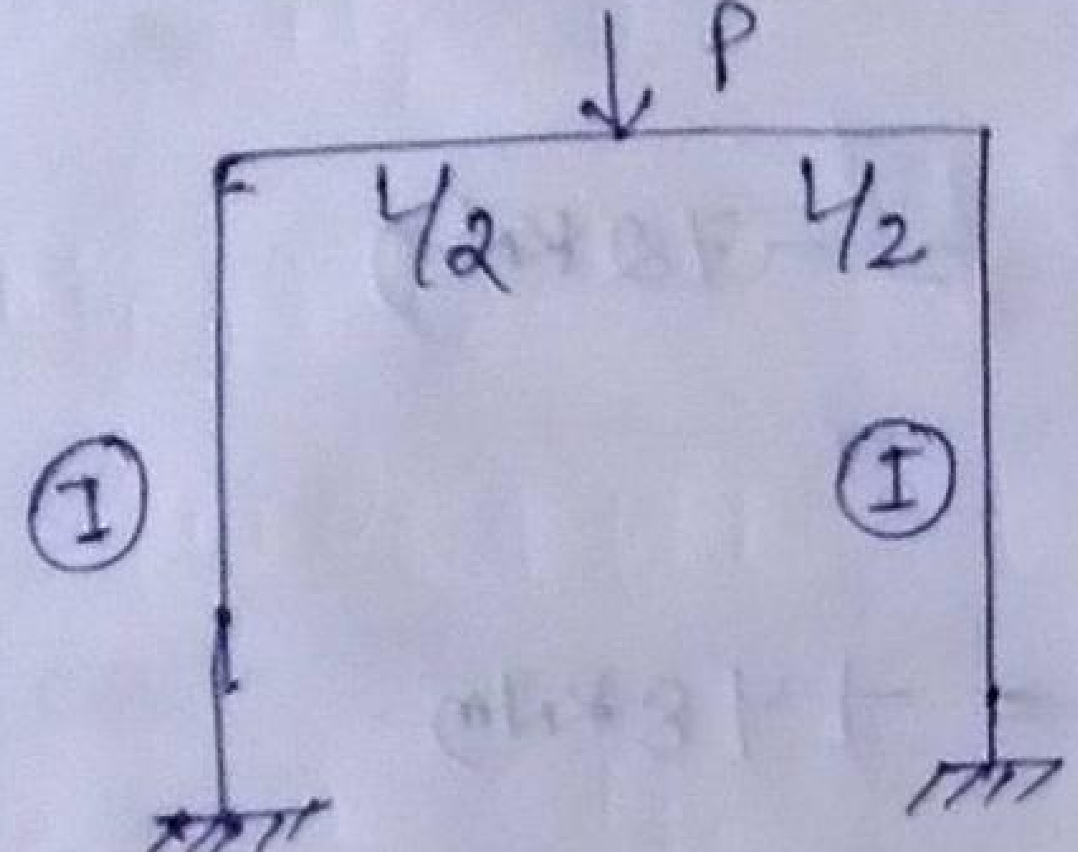
* Analysis of frames without sway:-

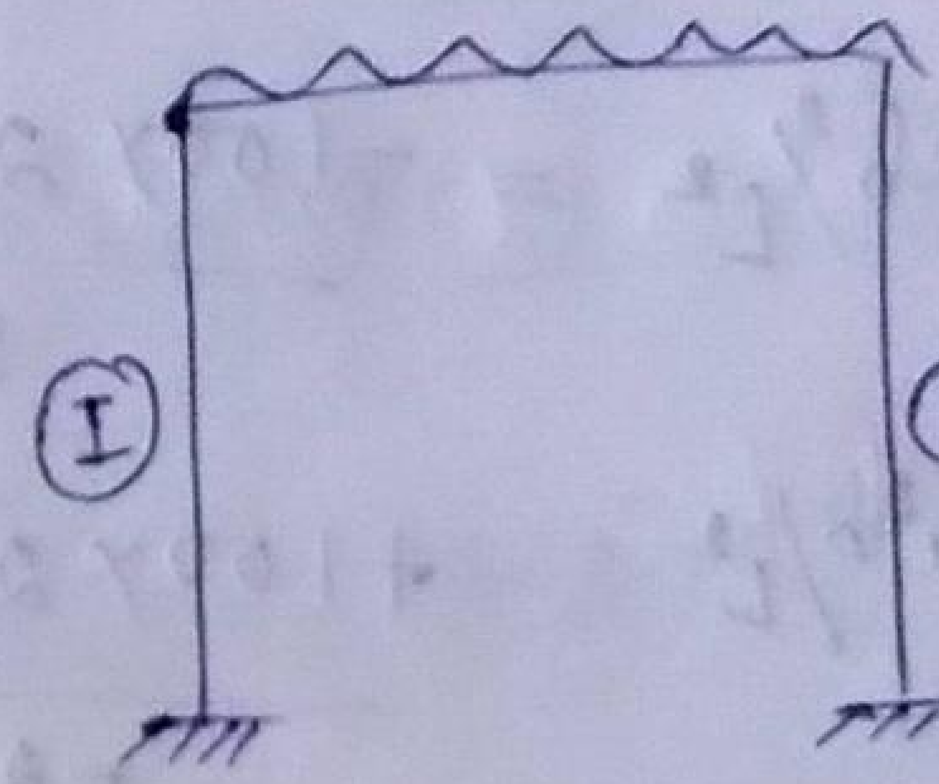
Frames don't undergo any sway when

(i) columns are at same level and net horizontal force is zero.

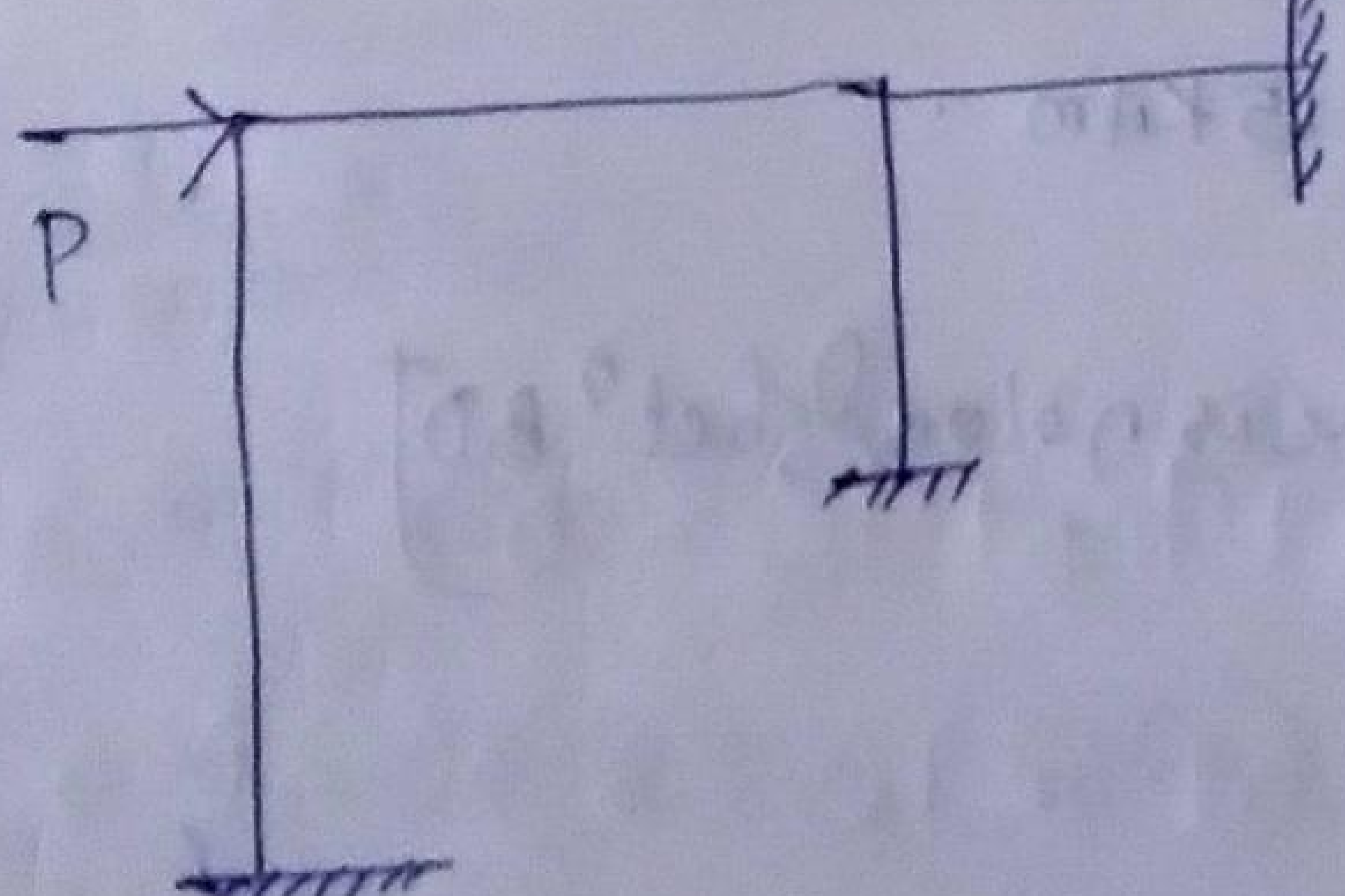


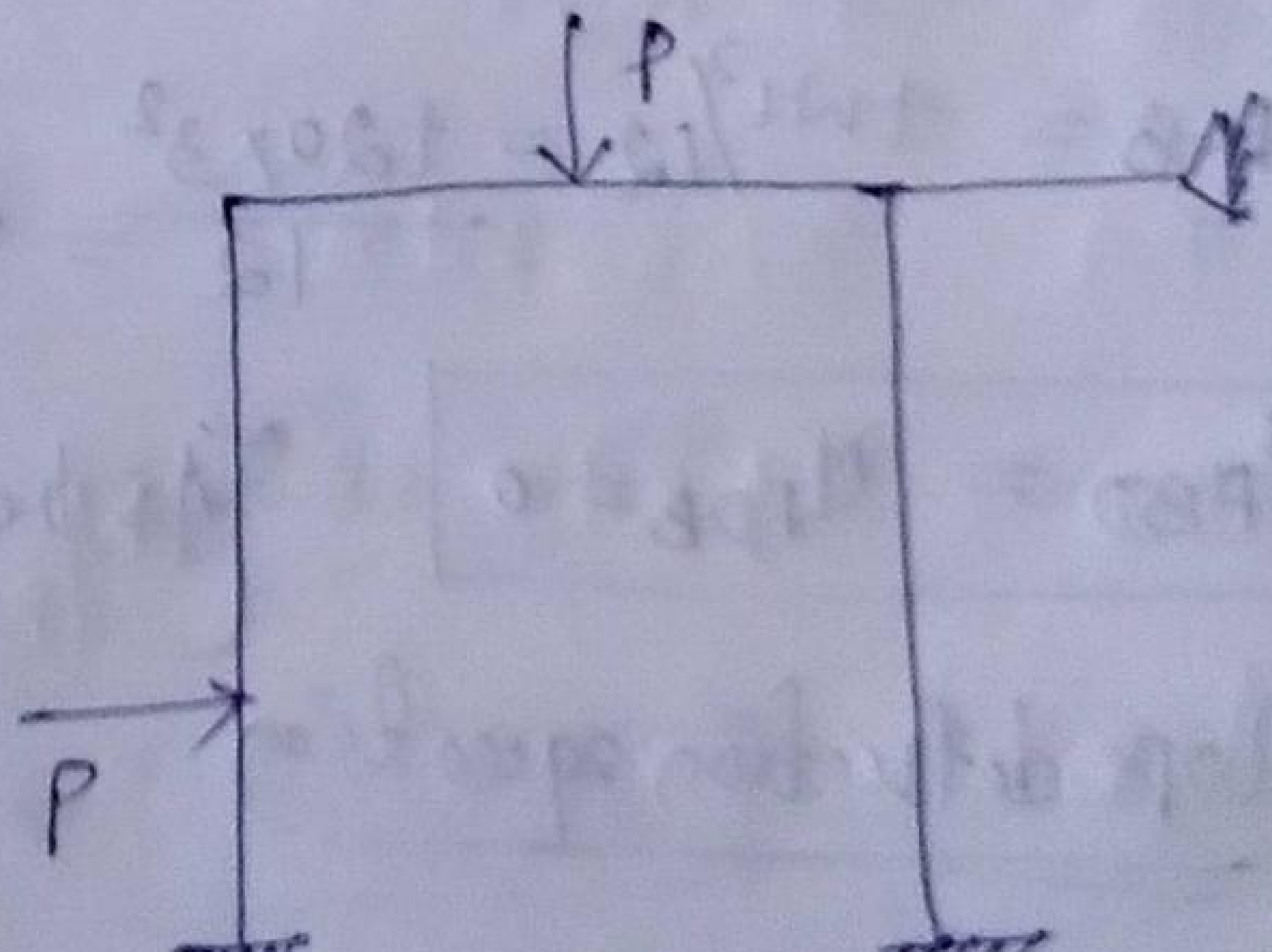
(ii) stiffness of column is same and loading is symmetrical in vertical plane.



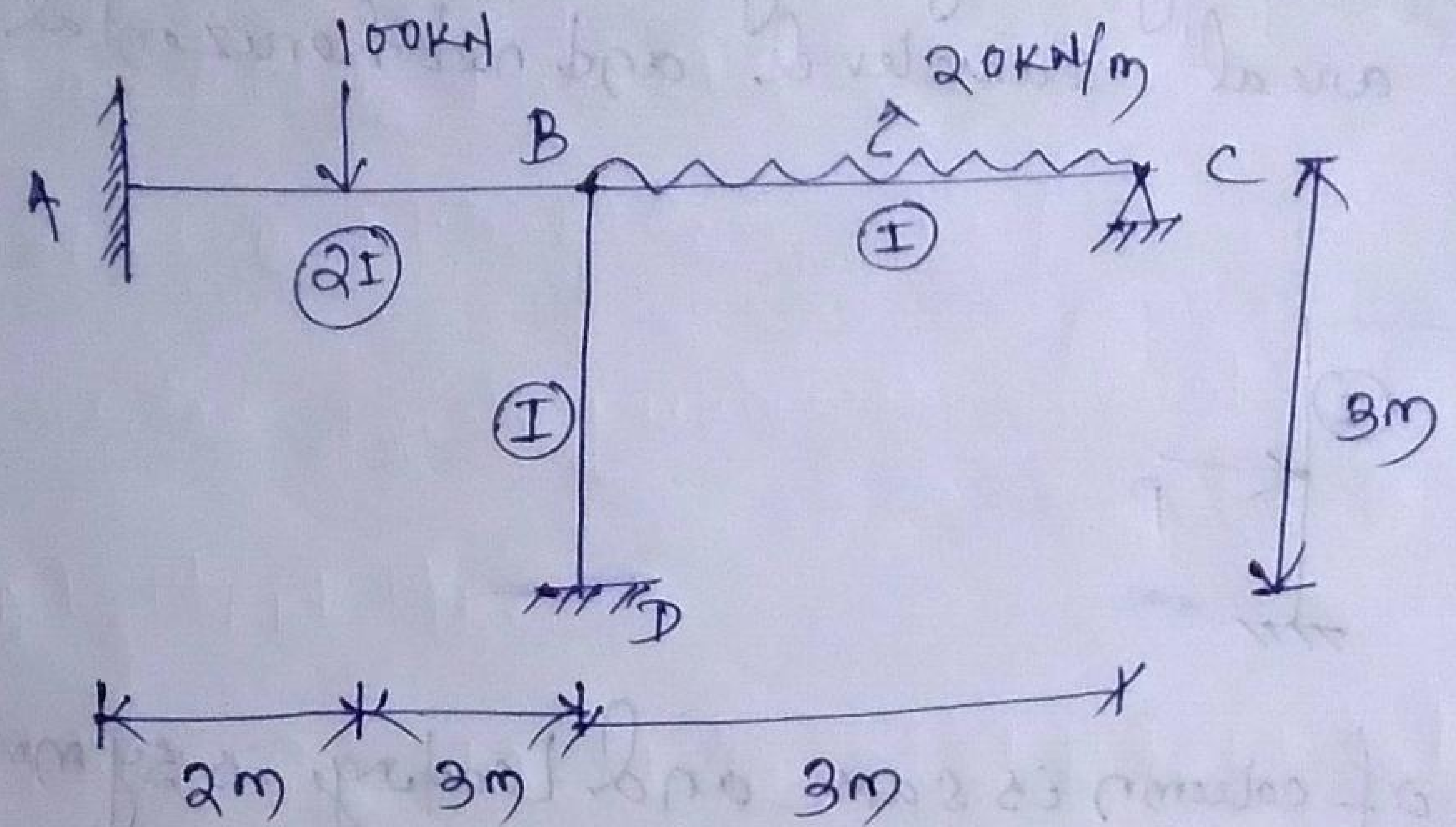


(iii) Sway are prevented by unyielding support at beam level.





Q Analyse the frame shown in figure below by slope deflection method. Draw the BMD.



Fixed end Moment.

$$M_{FAB} = -\frac{wab^2}{L^2} = \frac{-100 \times 2 \times 3^2}{5^2} = -72 \text{ KNm}$$

$$M_{FBA} = +\frac{wa^2b}{L^2} = \frac{+100 \times 2^2 \times 3}{5^2} = +48 \text{ KNm}.$$

$$M_{FBC} = -\frac{wL^2}{12} = \frac{-20 \times 3^2}{12} = -15 \text{ KNm}$$

$$M_{FCB} = +\frac{wL^2}{12} = \frac{+20 \times 3^2}{12} = +15 \text{ KNm}.$$

$$\boxed{M_{FBD} = M_{FDB} = 0}$$ [As there is no loads bet.ⁿ BD]

Slope deflection equation

$$M_{AB} = M_{FAB} + \frac{2E(2I)}{L}\left(2\theta_A + \theta_B - \frac{3\Delta}{L}\right)$$

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$$\Rightarrow -72 + \frac{4EI}{5}\left(2\cancel{\theta_A}^{0} + \theta_B - \cancel{3\Delta/L}^{0}\right)$$

$$\Rightarrow \boxed{-72 + \frac{4EI}{5}(\theta_B)} \quad \text{—— (1)}$$

$$M_{BA} = M_{FBA} + \frac{2E(2I)}{L}\left(2\theta_B + \cancel{\theta_A}^{0} - \cancel{3\Delta/L}^{0}\right)$$

$$= 48 + \frac{2E\times(2I)}{5}(2\theta_B)$$

$$= \boxed{48 + 1.6\,EI\theta_B} \quad \text{—— (2)}$$

Member BC

$$M_{BC} = M_{FBC} + 2EI/L\left(2\theta_B + \theta_C - \cancel{3\Delta/L}^{0}\right)$$

$$= \boxed{-15 + 2EI/3\,(2\theta_B + \theta_C)} \quad \text{—— (3)}$$

$$M_{CB} = M_{FCB} + 2EI/L\left(2\theta_C + \theta_B - \cancel{3\Delta/L}^{0}\right)$$

$$= 15 + 2EI/3\,(2\theta_C + \theta_B - 0)$$

$$= \boxed{15 + 0.67\,EI\theta_B + 1.33EI\theta_C} \quad \text{—— (4)}$$

Member BD :-

$$M_{BD} = M_{FBD} + 2EI/L\left(2\theta_B + \cancel{\theta_D}^{0} - \cancel{3\Delta/L}^{0}\right)$$

$$= 0 + 2EI/3\,(2\theta_B) = \boxed{1.33\,EI\theta_B} \quad \text{—— (5)}$$

$$M_{DB} = M_{FDB} + 2EI/3\left(2\cancel{\theta_D}^{0} + \theta_B - \cancel{3\Delta/L}^{0}\right)$$

$$= 0.67\,EI\theta_B \quad \text{—— (6)}$$

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Equilibrium equation

Since end 'c' is hinge, Hence $M_{CB} = 0$.

$\Rightarrow 15 + 0.67\,EI\theta_B + 1.33\,EI\theta_C = 0.$

$\Rightarrow \boxed{0.67\,EI\theta_B + 1.33\,EI\theta_C = -15}$ ——— (7)

Also consider joint equilibrium of joint B

$M_{BA} + M_{BD} + M_{BC} = 0$

$\Rightarrow 48 + 1.6\,EI\theta_B + 1.33\,EI\theta_B - 15 + 1.33\,EI\theta_B + 0.67\,EI\theta_C = 0$

$\Rightarrow \boxed{4.26\,EI\theta_B + 0.67\,EI\theta_C = -33}$ ——— (8)

On solving equation (7) and (8), we get

$EI\theta_B = -6.487$

$EI\theta_C = -8.01$

Final end Moments:-

$M_{AB} = -77.189$ KNm

$M_{BA} = 37.62$ KNm

$M_{BC} = -29$ KNm

$M_{BA} + M_{BD} + M_{BC} = 0$

$M_{CB} = 0$

$\Rightarrow 37.62 + (-8.63) + (-29) = 0.$

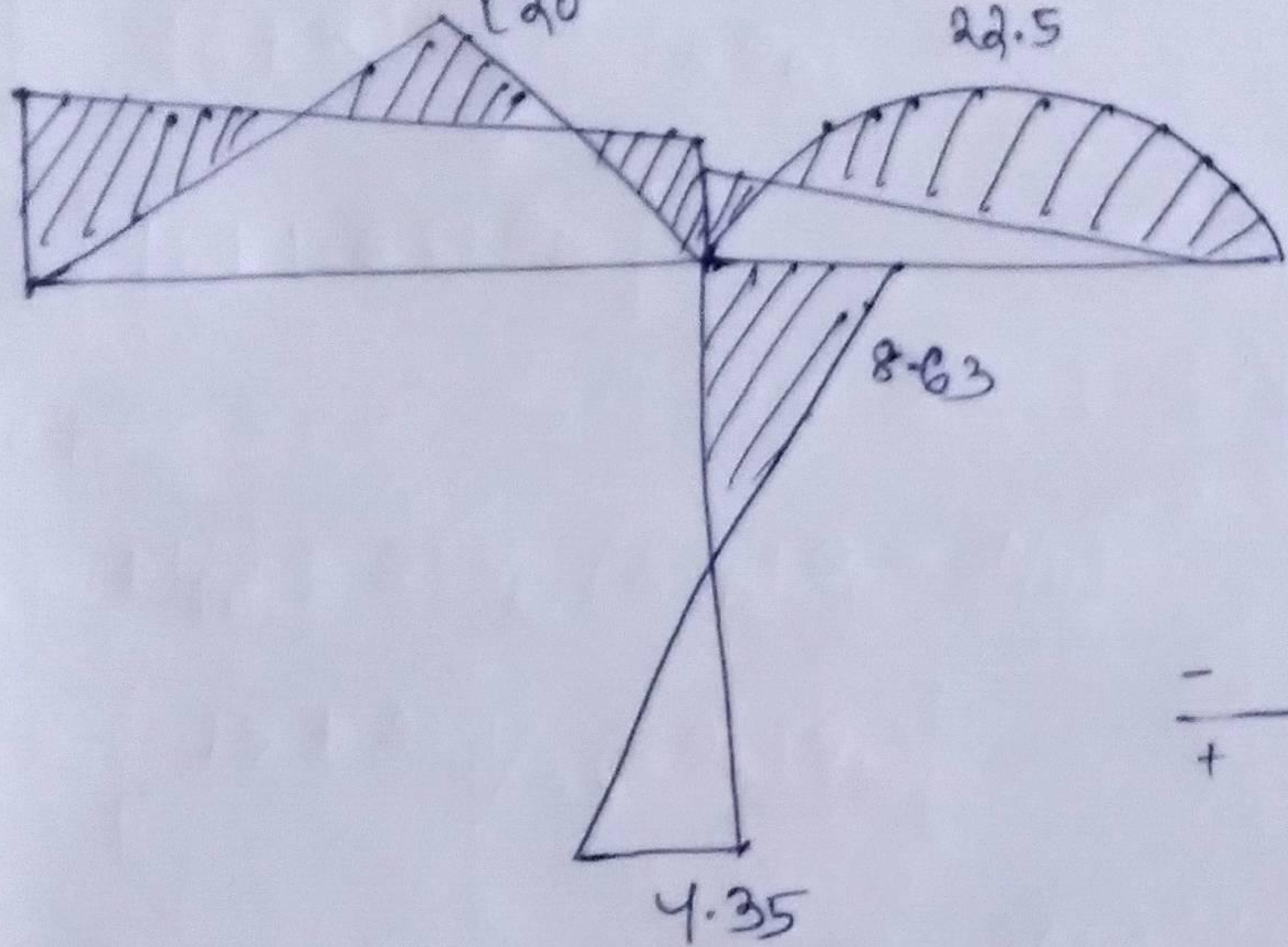
$M_{BD} = -8.63$ KNm

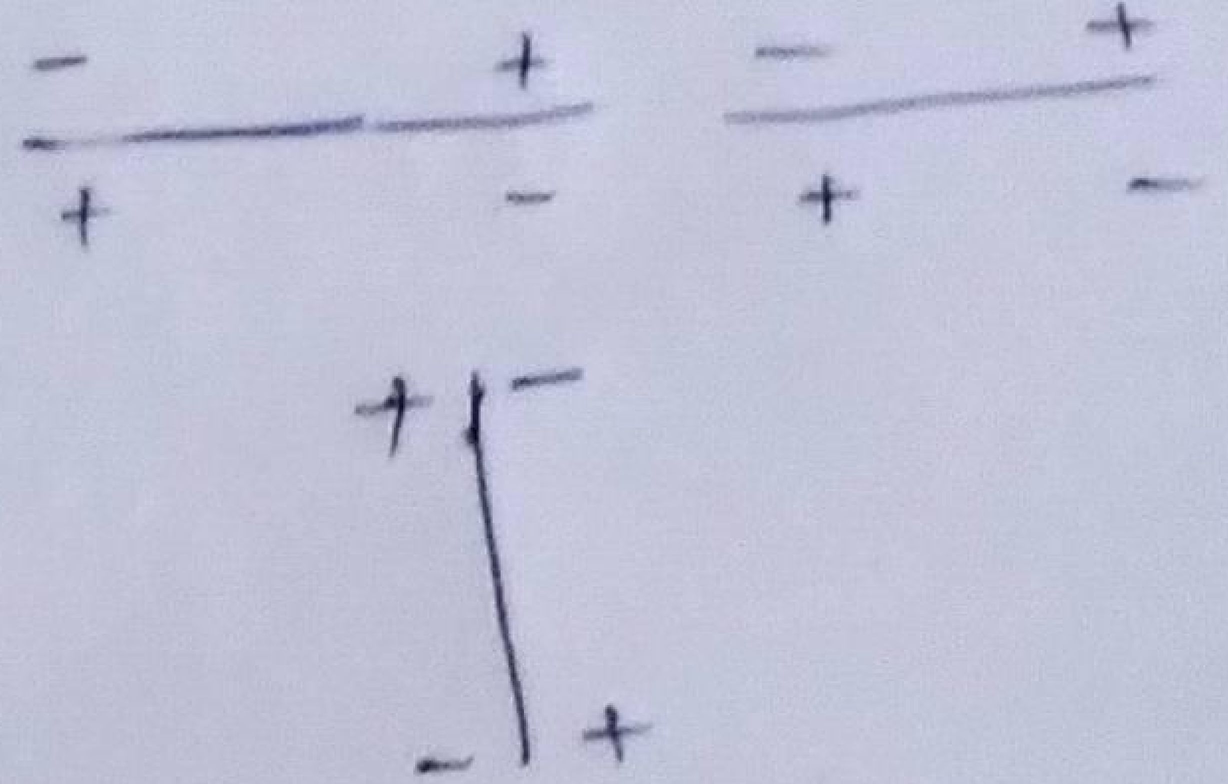
$M_{DB} = -4.35$ KNm

Free Bending Moment

$$M_{AB} = \frac{Wab}{L} = \frac{100 \times 2 \times 3}{5} = 120\,kNm$$

$$M_{BC} = WL^2/8 = 20 \times 3^2/8 = 22.5\,kNm.$$

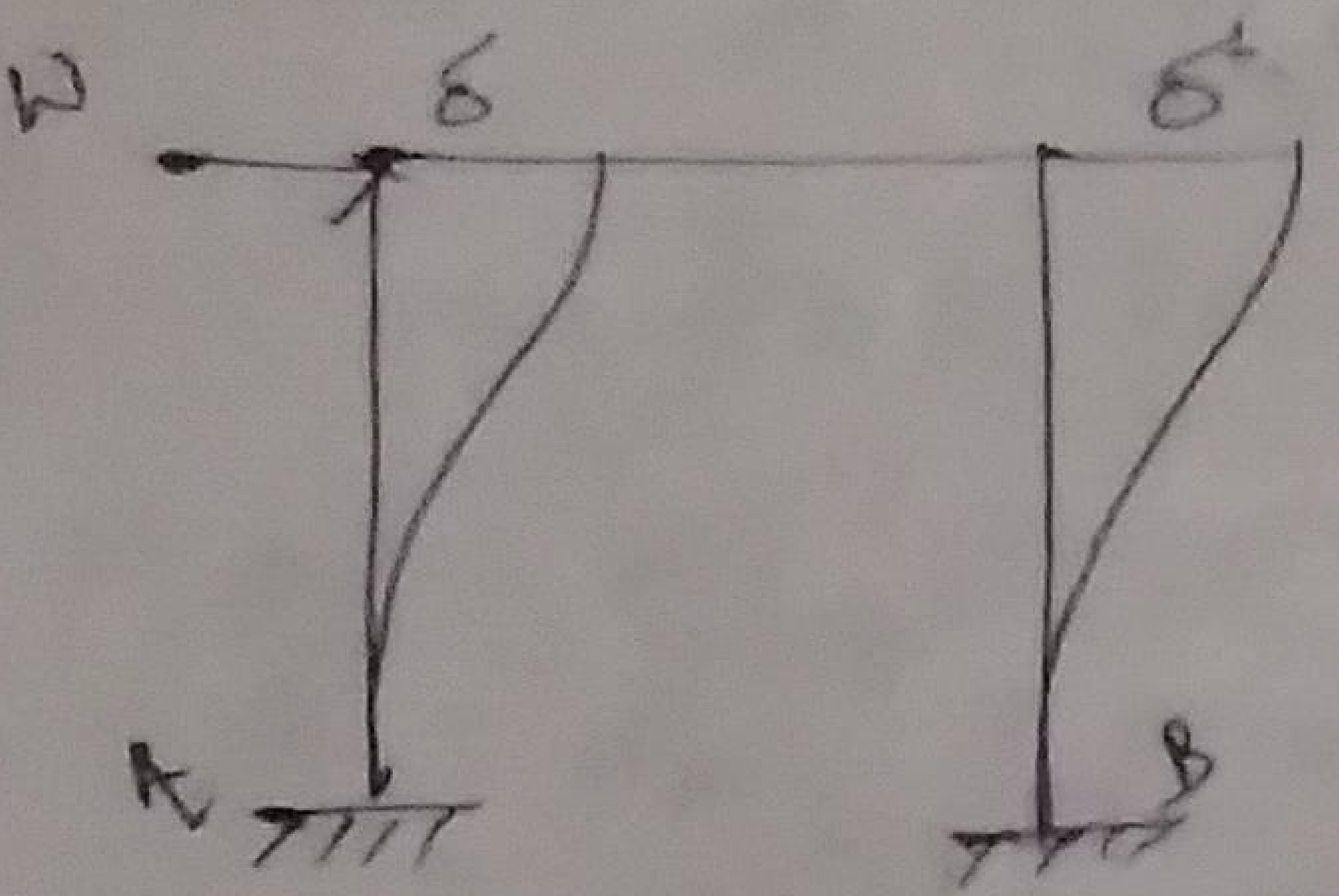




* Difference between sway frame and Non sway frame

<u>Sway frame</u>

→ The method of solving problem of sway frame is different

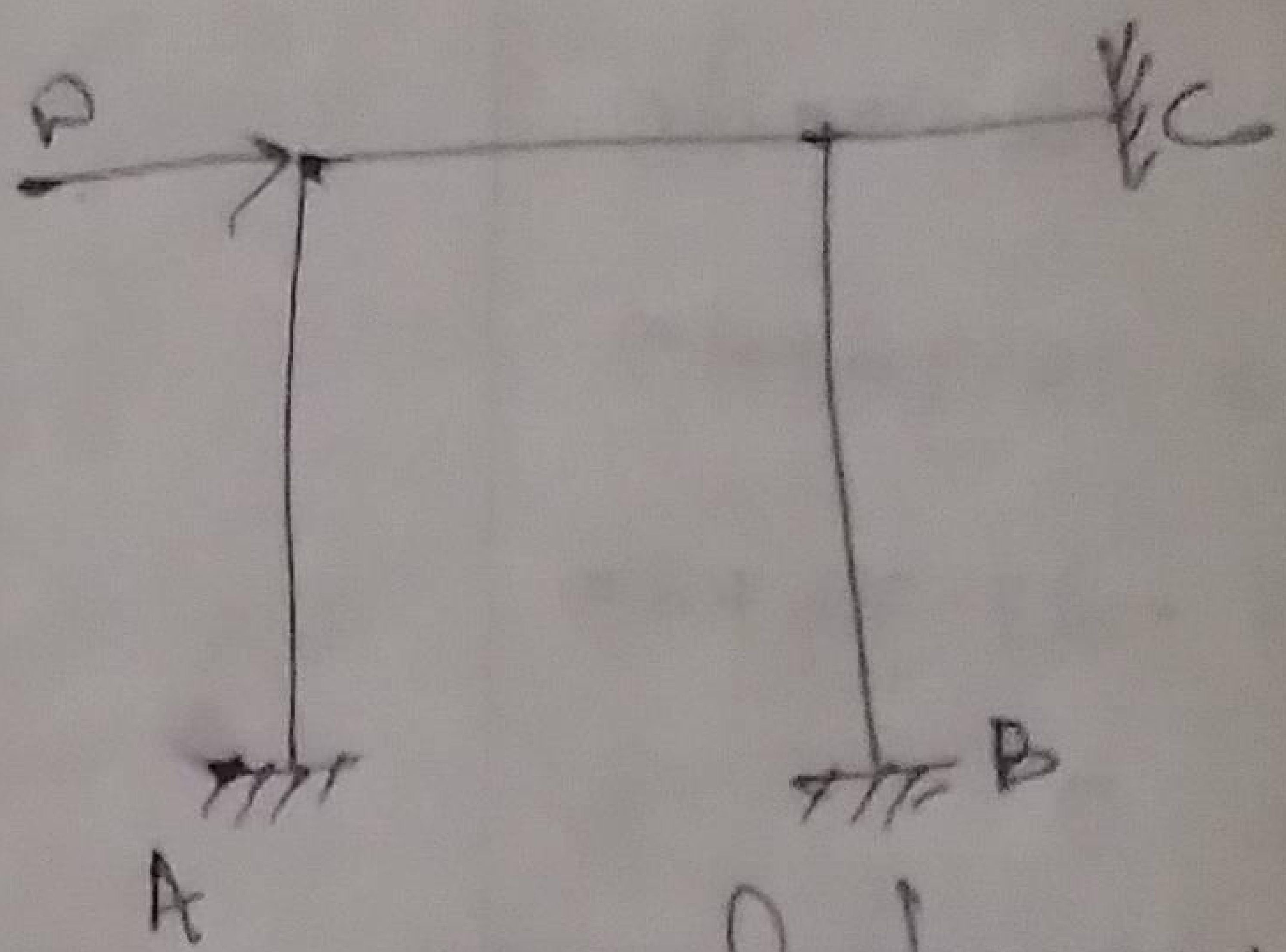


called sway due to unsymmetrical loading.

* If lateral deflection happens then that is sway frame.

<u>Non sway frame</u>

→ Solution of sway frame is same as beam.

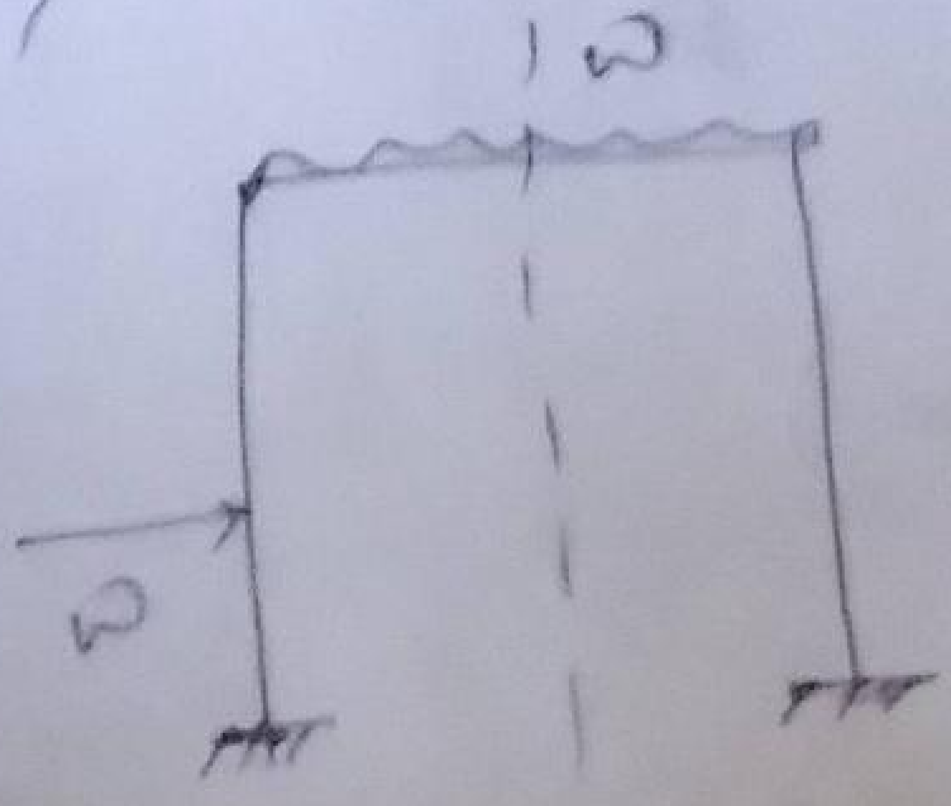


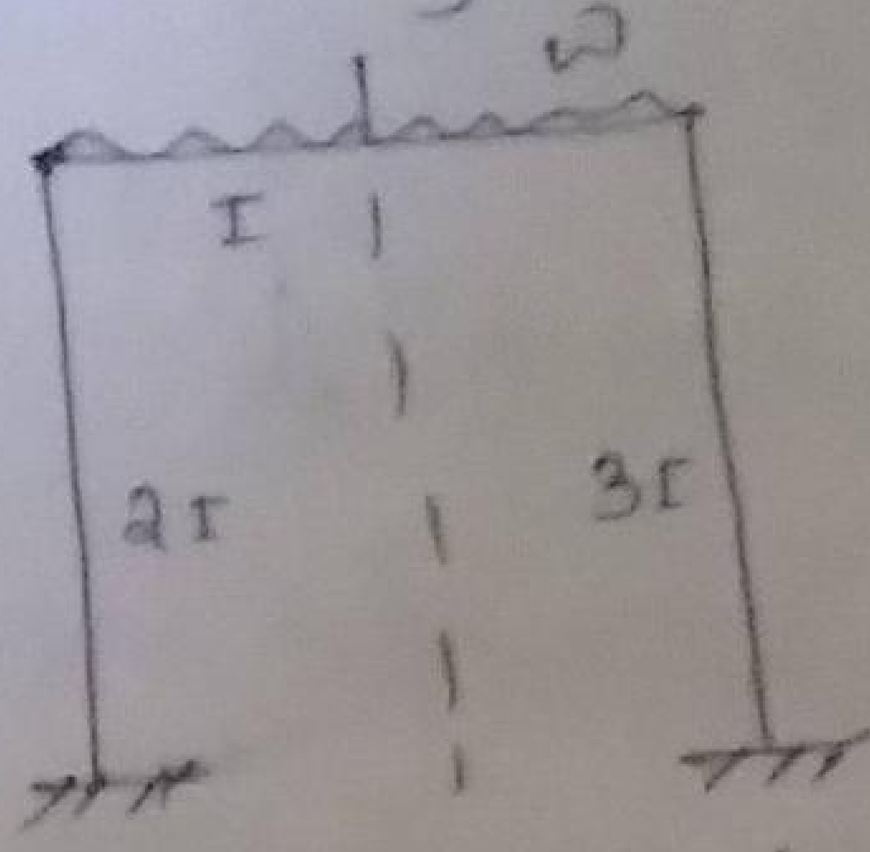
called Non sway because the deflection is resisted by support C

→ If no lateral deflection (non sway frame)

Conditions

1) check for symmetrical Loading





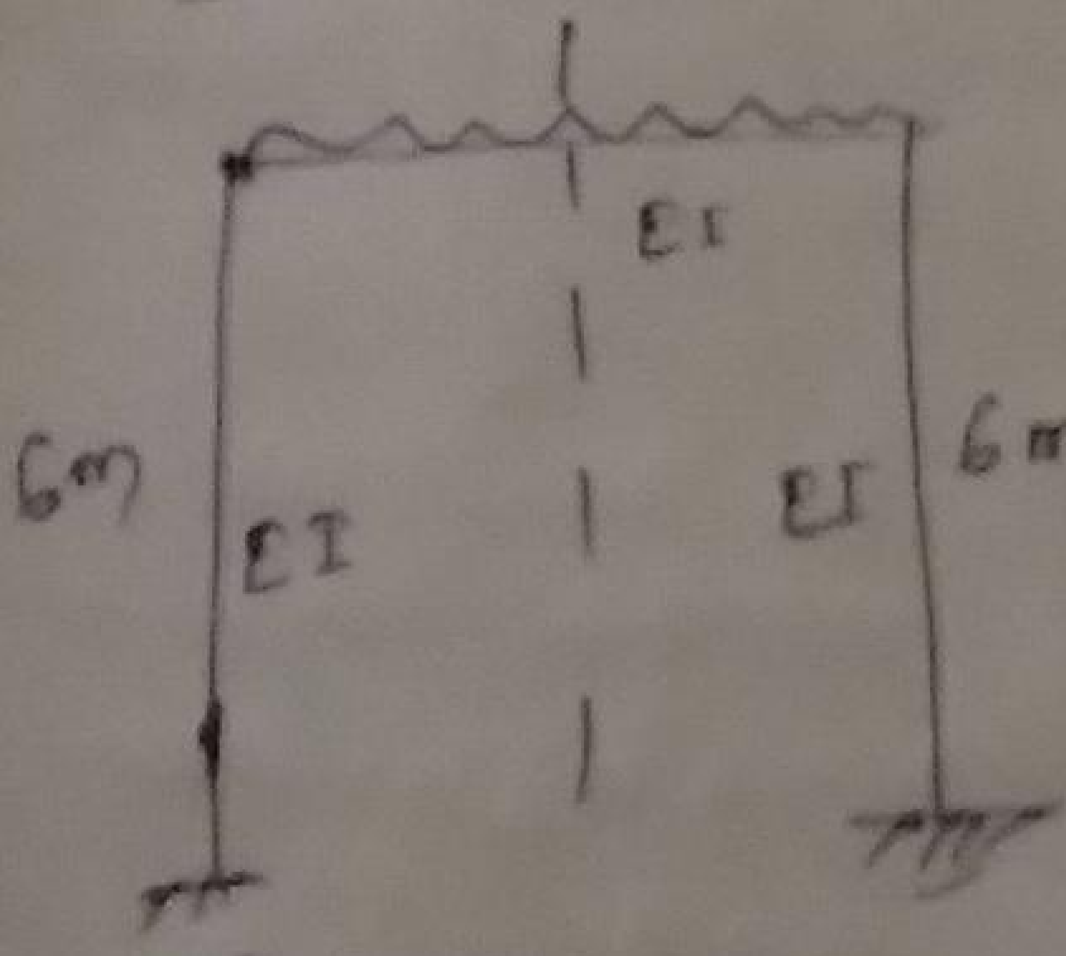
→ Sway frame.

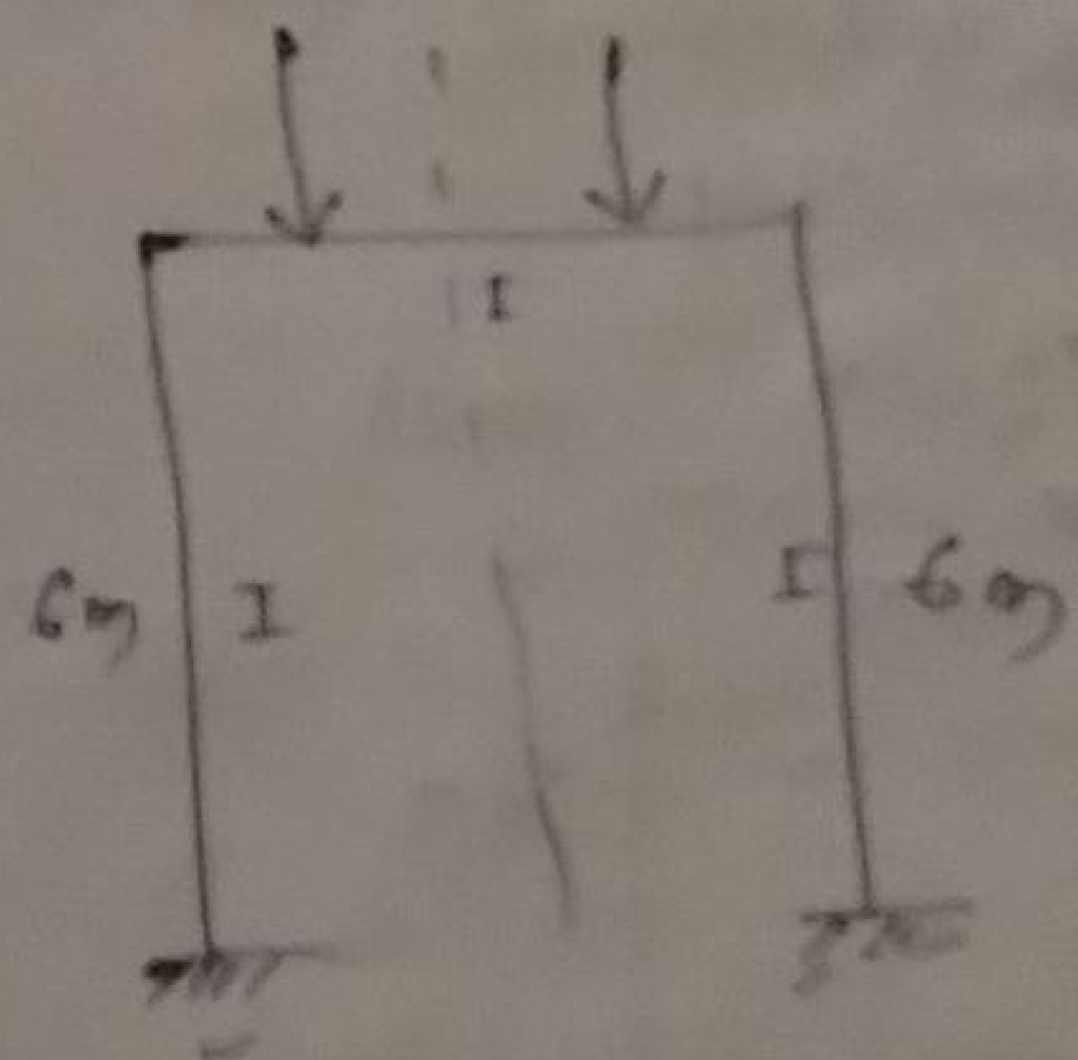
2) check for symmetrical EI given
3) check for vertical member length
4) check for symmetrical supports

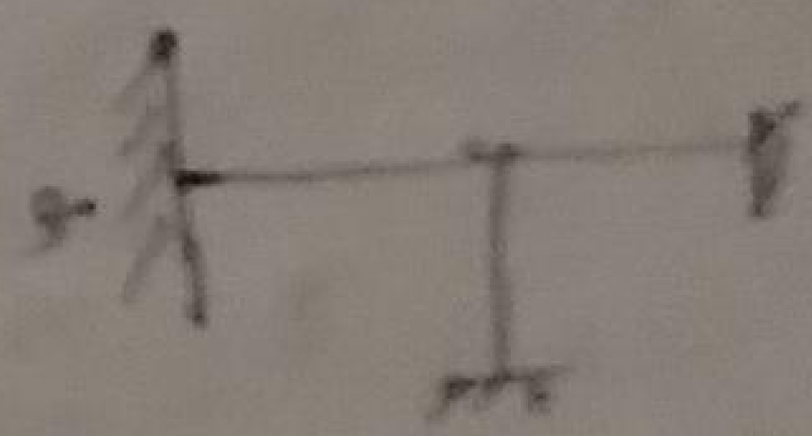
[* If above four conditions are satisfied then that is nonsway frame otherwise sway frame].

→ If only one condition not satisfied that is sway frame.

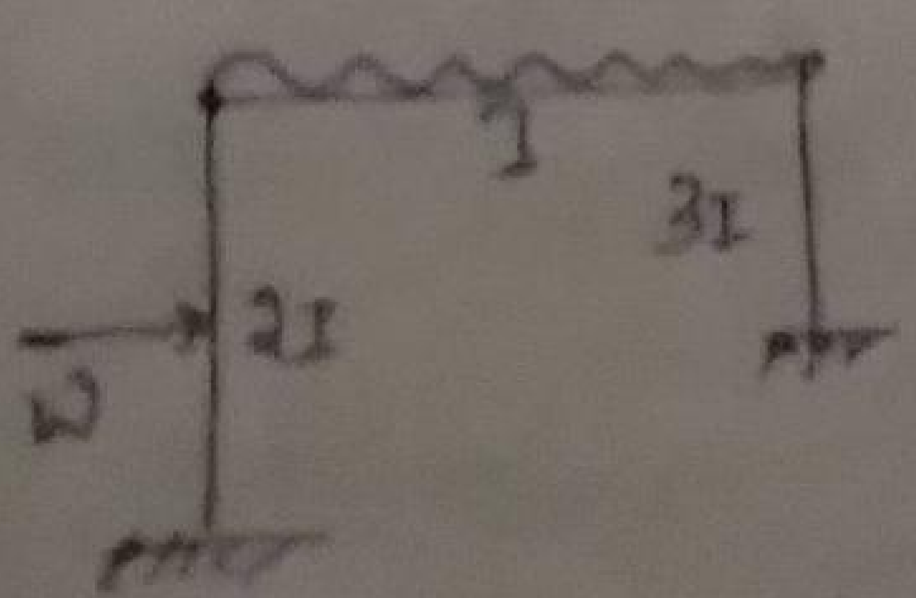
Non Sway

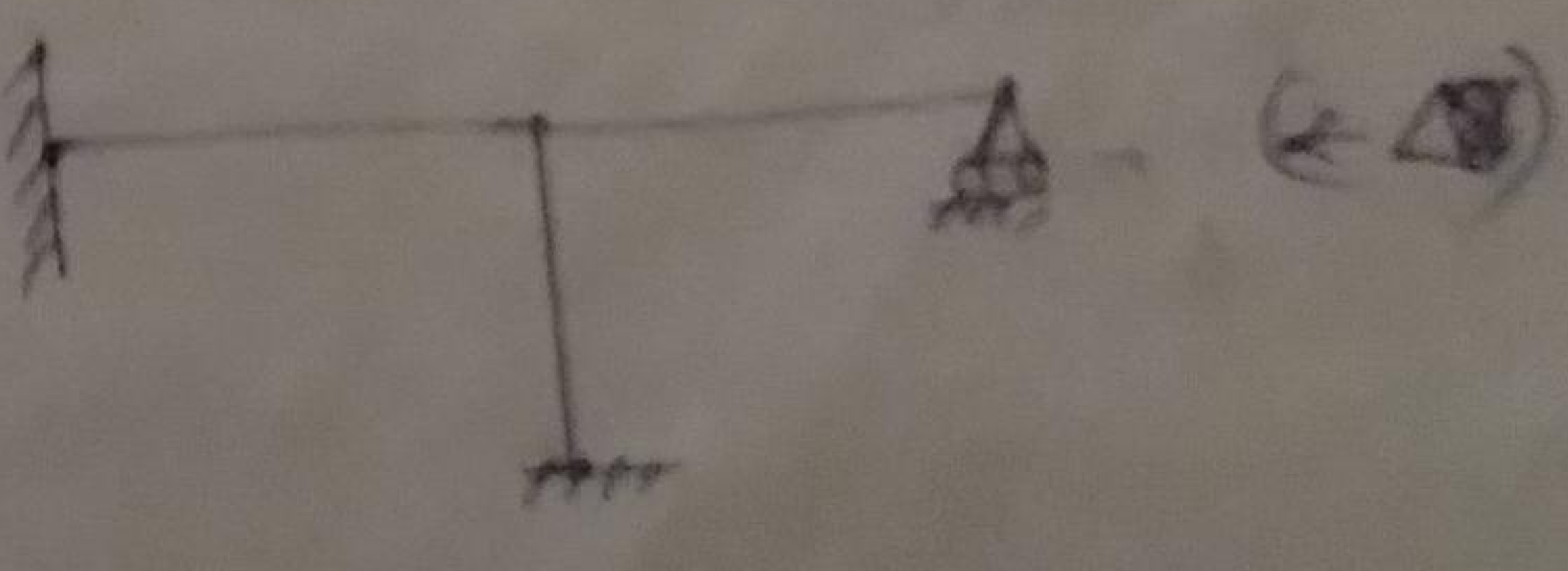


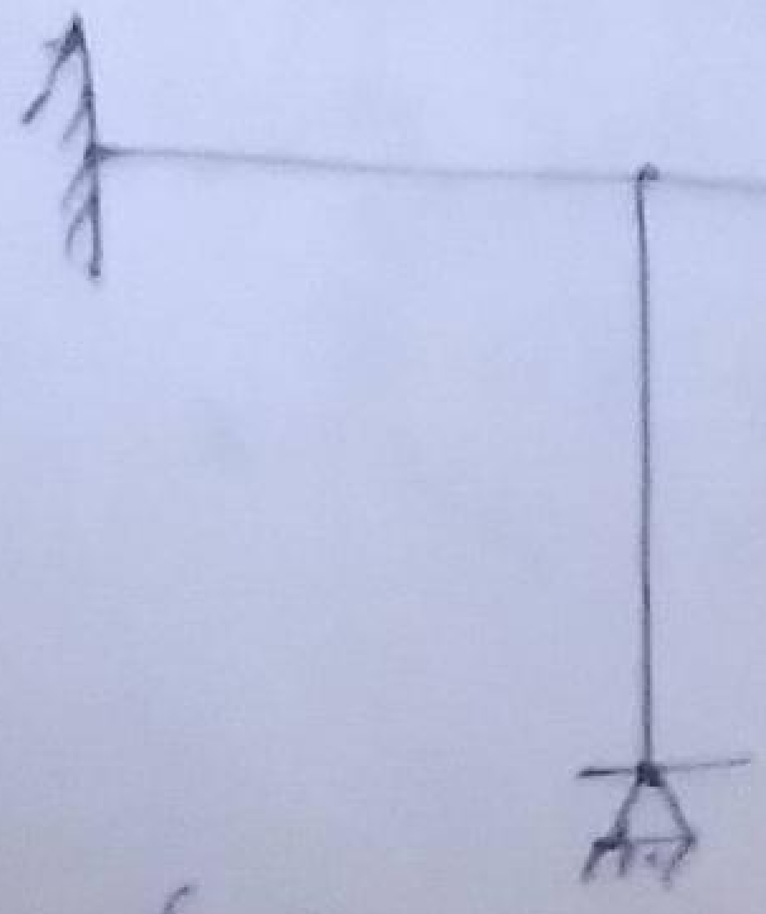




Sway

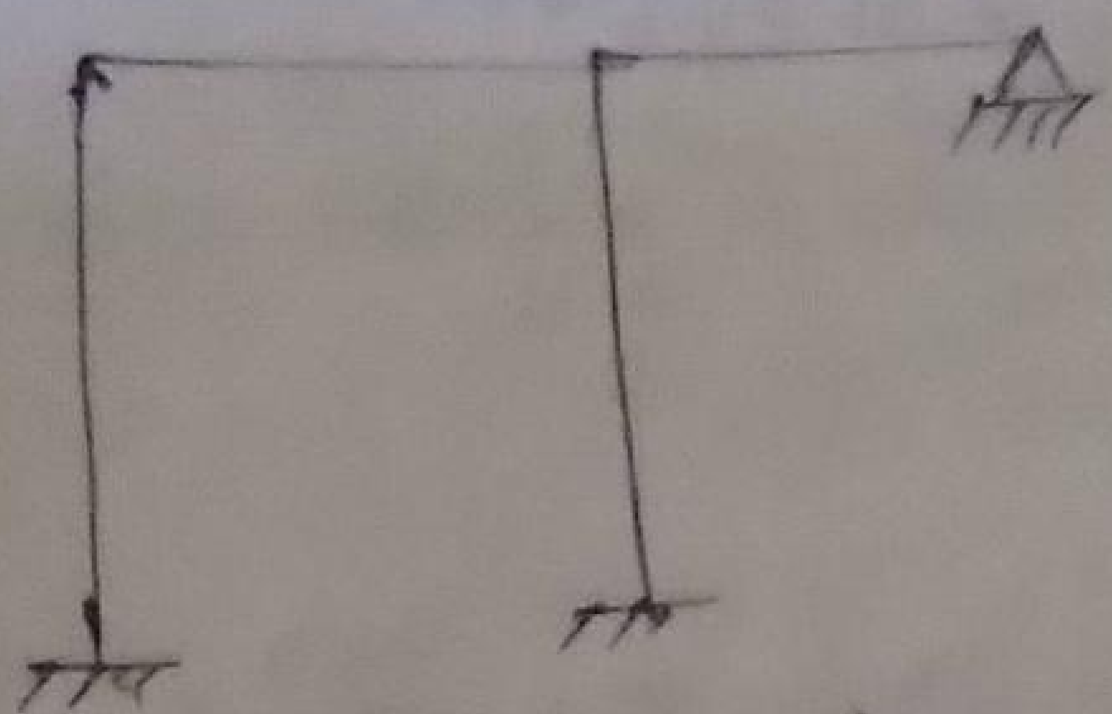




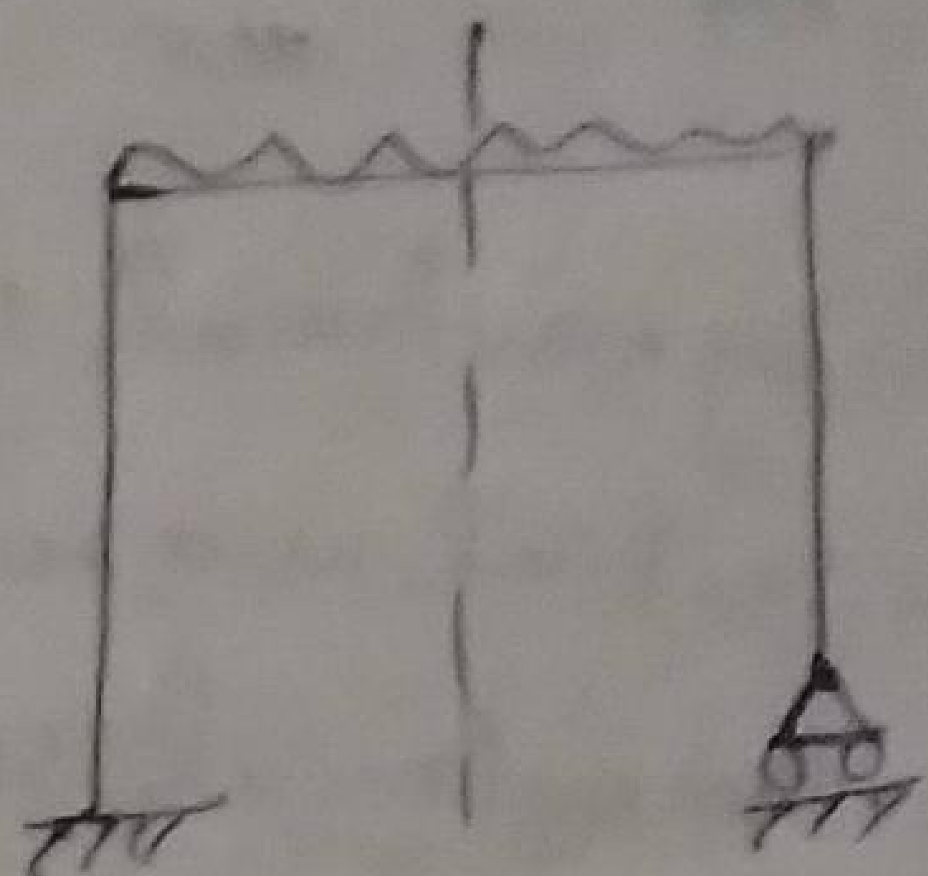


(Nonsway)

(Sway)



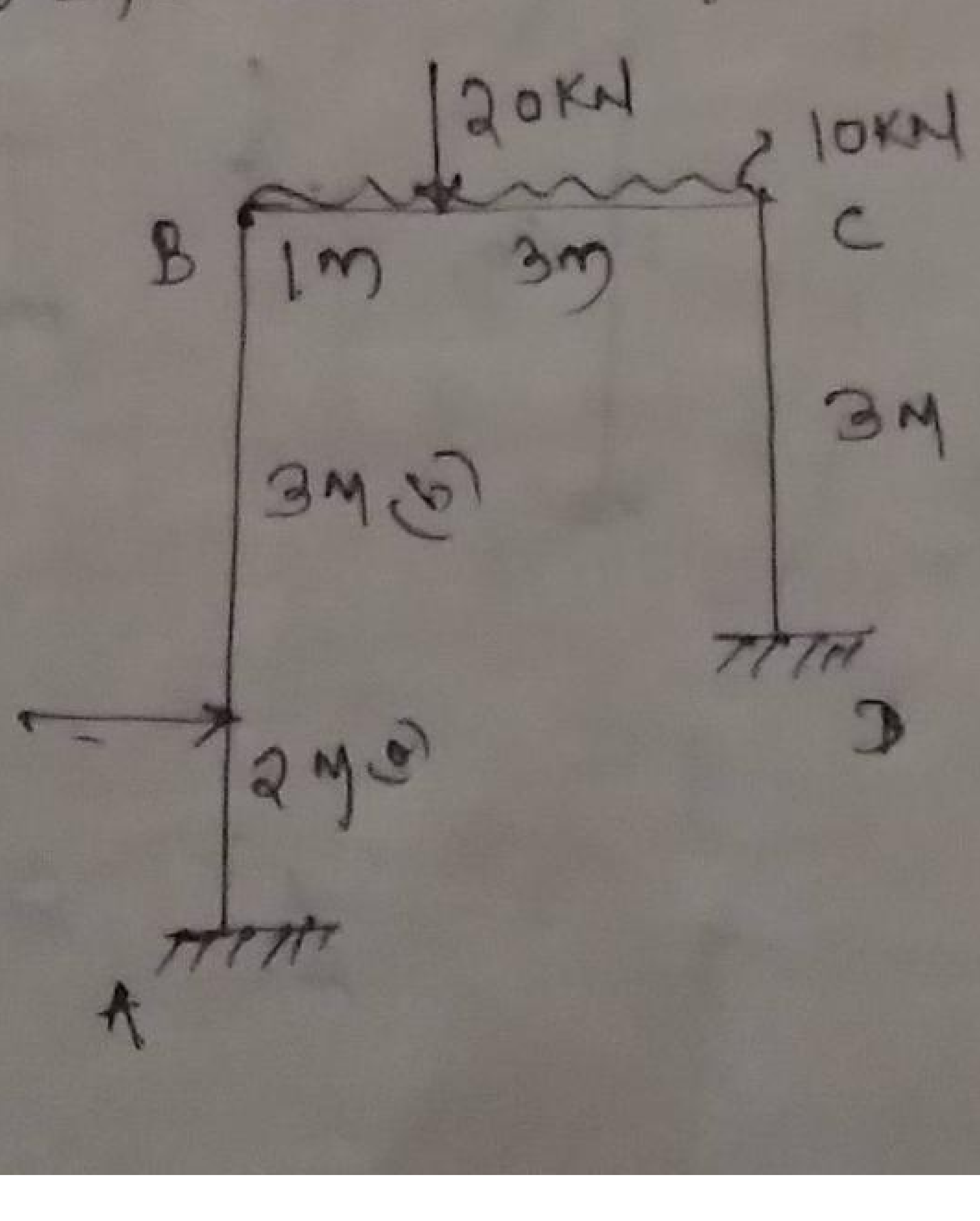
(Nonsway)



(Sway)

Problem on sway frame

Q using slope deflection method Analyse the frame loaded and supported as shown in figure and also draw BMD and deflected shape



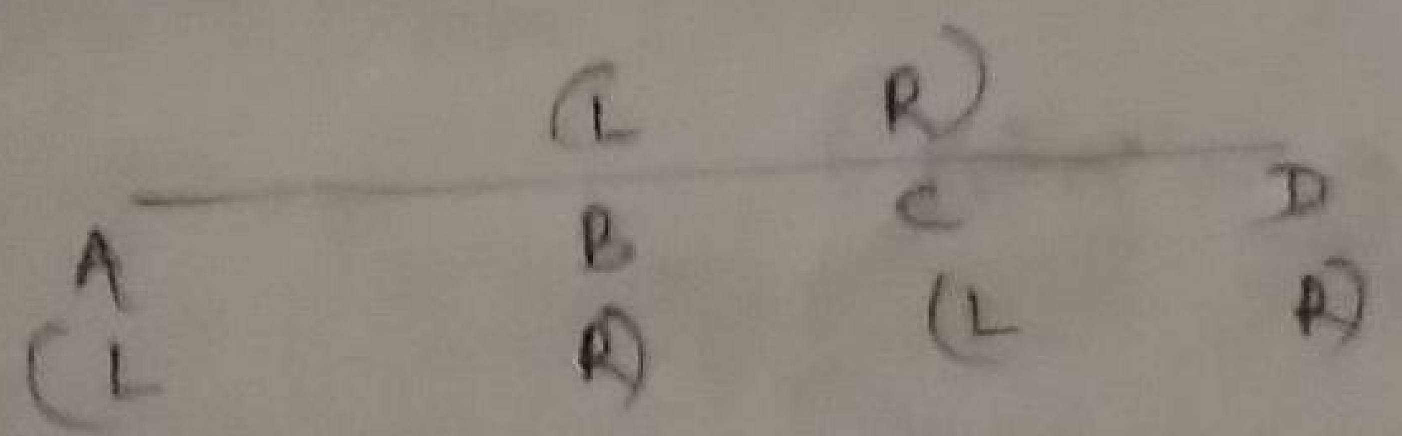
Conditions for sway frame

1) check for symmetrical loading
2) check for symmetrical EI is given
3) check for vertical member lengths
4) check for symmetrical supports

Note: While calculating fixed end moment consider all ends are fixed

and consider from left to right

Step-1



$$M_{FAB} = -wab^2/L^2 = \frac{-25 \times 2 \times 3^2}{5^2} = \boxed{-18\ KN/m}$$

$$M_{FBA} = +wa^2b/L^2 = \frac{25 \times 2^2 \times 3}{5^2} = \boxed{12\ KN/m}$$

M_{FBC} = for BC span we have to consider both udl & pt Load

$$= -\frac{wL^2}{12} \cancel{\frac{wL}{8}} - wab^2/L^2$$

$$= \frac{-10 \times 4^2}{12} - \frac{20 \times 1 \times 3^2}{4^2}$$

$$= \boxed{-24.58\ KN/m}$$

$$M_{FCB} = +wL^2/12 + wa^2b/L^2$$

$$= 10 \times 4^2/12 + \frac{20 \times 1^2 \times 3}{4^2} = \boxed{17.08\ KN/m}$$

$M_{FCD} = 0$ (as no load) (left to right)

$M_{FDC} = 0$ (Right to left)

Step-2: Apply slope and deflection equation for each span

$M_{AB} = M_{FAB} + 2EI/L\,(2\theta_A + \theta_B - 3\Delta/L)$

[Note: In sway frame deflection is only in vertical member. Deflection never act on horizontal member]
→ AB is vertical member and CD also.

$$M_{AB} = -18 + \frac{2EI}{5}\left(2\cancel{\theta_A}^{\,0} + \theta_B - 3\Delta/5\right)$$

$$= \boxed{-18 + \frac{2}{5}EI\,\theta_B - 0.24\Delta} \text{ ——— (1)}$$

[∵ At fixed end as slope is zero so $\theta_A = 0$. When value of EI is not given that means that is constant] ($EI = 1$)

$M_{BA} = M_{FBA} + 2EI/L\,(2\theta_B + \theta_A - 3\Delta/L)$

$$= 12 + \frac{2}{5}(2\theta_B + 0 - 3\Delta/5)$$

$$= \boxed{12 + 4/5\,\theta_B - 0.24\Delta} \text{ ——— (2)}$$

$M_{BC} = M_{FBC} + 2EI/L\,(2\theta_B + \theta_C - \cancel{3\Delta/L}^{\,0})$ [for horizontal member there is no displacement so $\Delta = 0$]

$$= -24.58 + \frac{2EI}{4}(2\theta_B + \theta_C)$$

$$= \boxed{-24.58 + \theta_B + \theta_C \cdot 2/4} \text{ ——— (3)}$$

$M_{CB} = M_{FCB} + 2EI/L\,(2\theta_C + \theta_B - \cancel{3\Delta/L}^{\,0})$ [∵ BC also horizontal]

$$M_{CB} = 17.08 + \frac{2}{4}(2\theta_C + \theta_B)$$

$$= \boxed{17.08 + \frac{2}{4}\theta_B + \theta_C} \quad \text{——— (4)}$$

$$M_{CD} = M_{FCD} + 2EI/L\,(2\theta_C + \theta_D - 3\Delta/L)$$

$$= 0 + \frac{2EI'}{3}(2\theta_C + 0 - 3\Delta/L)$$

[as θ_D is fixed so (0)]

$$= \boxed{4/3\,\theta_C - 2/3\,\Delta} \quad \text{——— (5)}$$

$$M_{DC} = M_{FDC} + 2EI/L\,(2\theta_D + \theta_C - 3\Delta/L)$$

$$= 0 + \frac{2}{3}(2\times 0 + \theta_C - 3\Delta/3)$$

$$= \boxed{\frac{2'}{3}\theta_C - 2/3\,\Delta} \quad \text{——— (6)}$$

Note: for each member no. of equations = 2 (Left to right & Right to left)
for 3 " = 6 equations.

Step: 3 :- Apply condition of equilibrium :-

$\boxed{M_{BA} + M_{BC} = 0}$ ⇒ for B Joint → (1)

$\boxed{M_{CB} + M_{CD} = 0}$ ⇒ for 'C' Joint → (2)

$$M_{BA} + M_{BC} = 0$$

$$\Rightarrow [12 + 4/5\,\theta_B - 0.24\Delta] + [(-24.58 + \theta_B + 2/4\,\theta_C] = 0$$

$$\Rightarrow \boxed{\tfrac{4}{5}+1\,\theta_B+\tfrac{2}{4}\,\theta_C-0.24\,\Delta=-12+24.58} \text{ --- } (1)$$

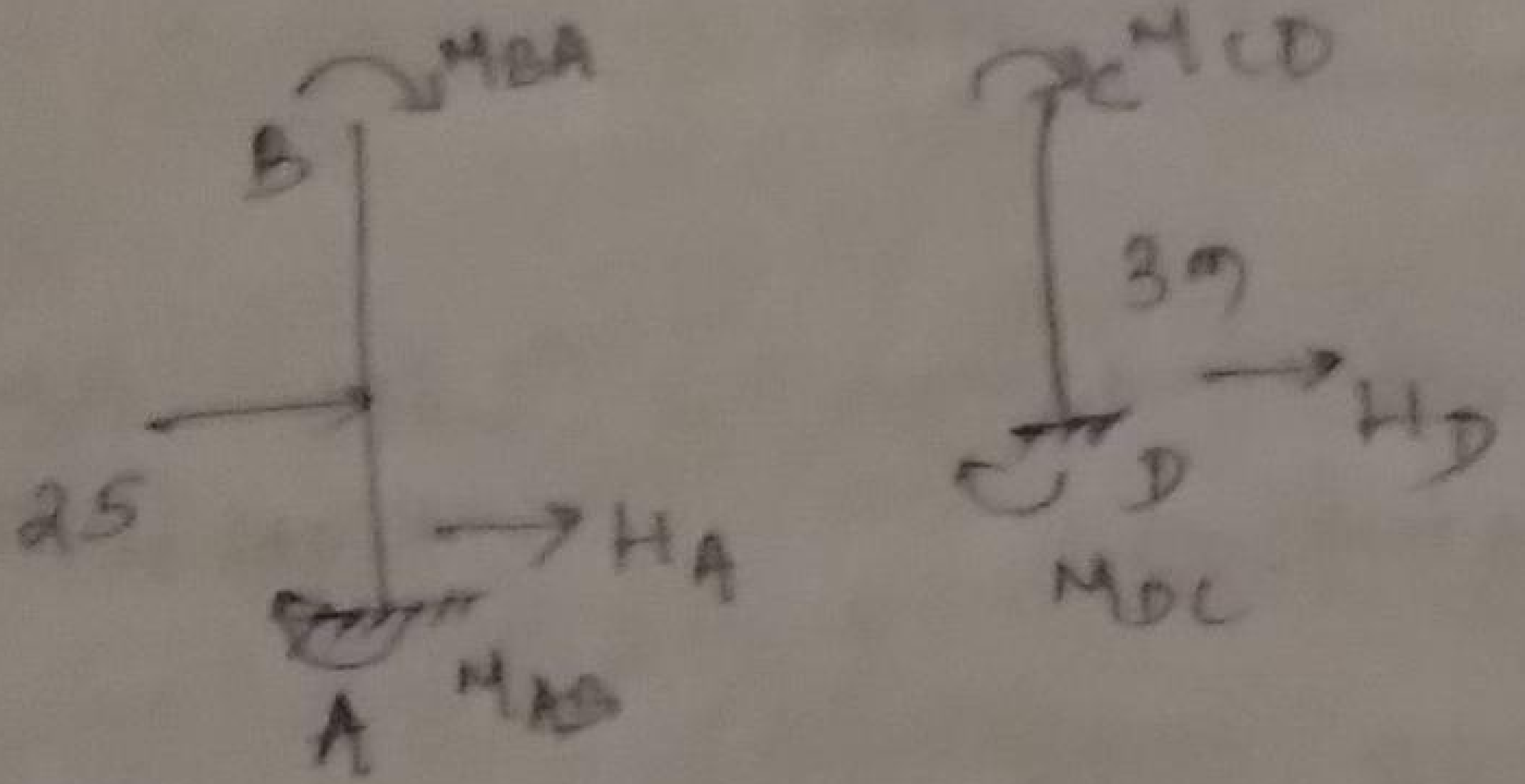
and $M_{CB}+M_{CD}=0$

$$\Rightarrow 17.08+\tfrac{2}{4}(\theta_B+\theta_C)+\left[\tfrac{4}{3}\theta_C-\tfrac{2}{3}\Delta\right]=0$$

$$\Rightarrow \boxed{\tfrac{2}{4}\,\theta_B+\tfrac{4}{3}+\theta_C-\tfrac{2}{3}\,\Delta=-17.08} \text{ --- } (2)$$

Here 3 unknowns such as θ_B, θ_C and Δ so we have to consider 3 equation. but till now we have 2 equation. (1 equation needed)

Calculate horizontal equilibrium for 1 more equation



Note
All moments are considered as +ve (Clockwise)
and forces are also +ve left to right

for member BA

$\Sigma M_B=0$ (+ clockwise)

$$M_{AB}+M_{BA}-(25\times 3)-H_A\times 5=0$$

$$\Rightarrow M_{AB}+M_{BA}-75=H_A\times 5$$

$$\Rightarrow \boxed{H_A=\frac{M_{AB}+M_{BA}-75}{5}}$$

or formula (1)

$$H_A=\frac{M_{AB}+M_{BA}-Pa}{L_{AB}}$$

for member CD

$\Sigma M_C = 0 \quad \circlearrowright +$

$\Rightarrow M_{DC} + M_{CD} - H_D \times 3 = 0$

$\boxed{H_A + H_D + P = 0}$ (3)

formula

$\Rightarrow \boxed{H_D = \frac{M_{DC} + M_{CD}}{3}}$ and $\boxed{H_D = \frac{M_{DC} + M_{CD}}{L_{CD}}}$ (2)

for horizontal equilibrium $\left[\Sigma F_x = 0 \quad \xrightarrow{(+)} \quad \xleftarrow{(-)}\right.$

$\boxed{H_A + H_D + 25 = 0}$

$\Rightarrow \left(\frac{M_{AB} + M_{BA} - 75}{5}\right] + \left[\frac{M_{DC} + M_{CD}}{3}\right] = 0$

$\Rightarrow \left[\frac{[(-18 + 2/3\,\theta_B - 0.24\Delta] + [(12 + 4/5\,\theta_B - 0.24\Delta] - 75}{5}\right.$

$+ \left[\frac{(2/3\,\theta_C - 2/3\,\Delta] + (4/3\,\theta_C - 2/3\,\Delta)}{3}\right] + 25 = 0$

separate θ_B first and then θ_C

$\Rightarrow$

$\theta_B = \left[\frac{2/5}{5} + \frac{4/5}{5}\right] = 0.24\theta_B \qquad \Delta = \left[-\frac{0.24}{5} - \frac{0.24}{5}\right.$

$\theta_C = \left[\frac{2/3}{3} + \frac{4/3}{3}\right] = 0.67\theta_C \qquad \left. -\frac{2/3}{3} - \frac{2/3}{3}\right]$

$\boxed{\theta_B = 0.24\,\theta_B}$

$\boxed{\Delta = -0.54\Delta}$

$\boxed{\theta_C = 0.67\theta_C}$

$\Rightarrow 0.24\theta_B + 0.67\theta_C - 0.51\Delta = \frac{18}{5} - \frac{12}{5} + \frac{75}{5} - 25$

$\Rightarrow \boxed{0.24\theta_B + 0.67\theta_C - 0.51\Delta = -8.8}$ ——— (3)

Solving eqⁿ (1), (2) & (3)

$\theta_B = 10.40$

$\theta_C = -5.53$

$\Delta = 14.05$

Step 4 find Final Moments

put $\theta_B, \theta_C, \Delta$ in eqⁿ of slope and deflection (1 to 6)

$M_{AB} = -17.21$ kNm

$M_{BA} = 16.94$ kNm

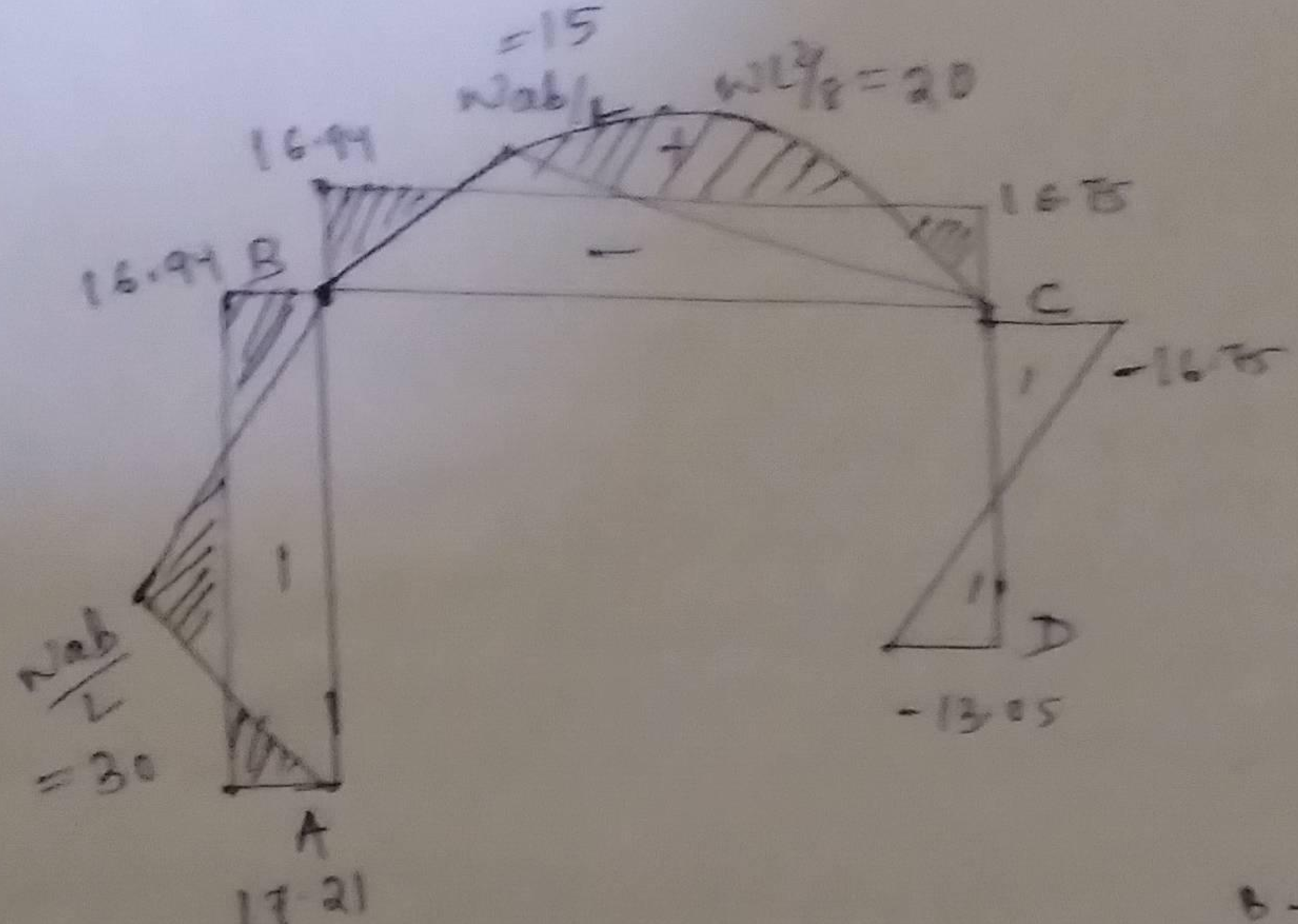
$M_{BC} = -16.94$ kNm

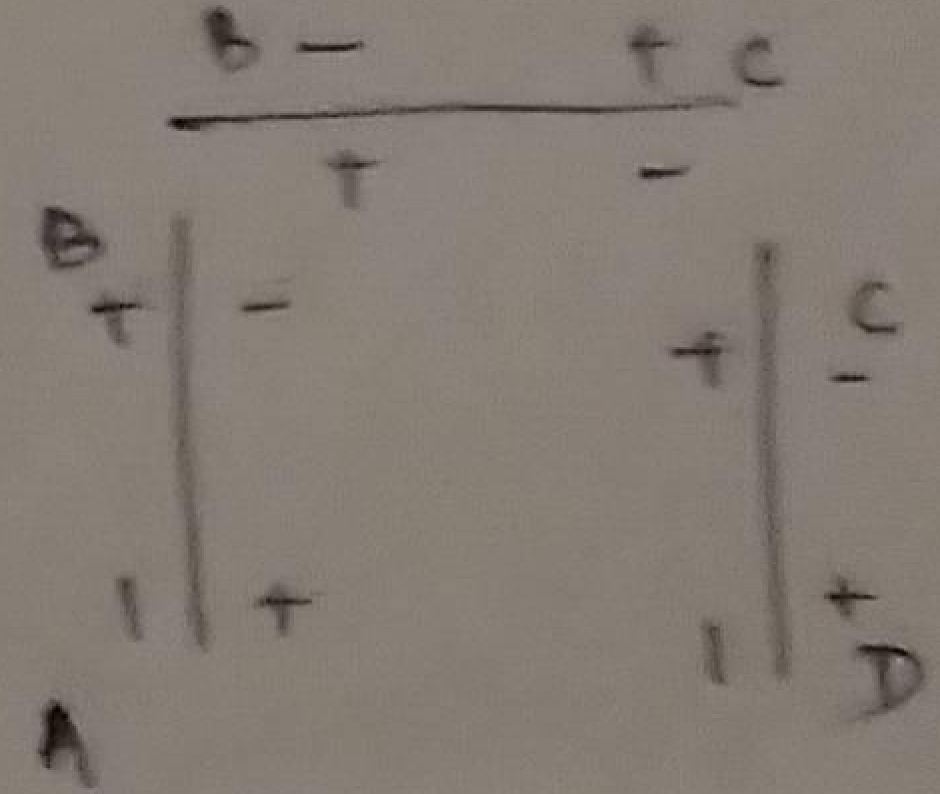
$M_{CB} = 16.75$ kNm

$M_{CD} = -16.75$ kNm

$M_{DC} = -13.05$ kNm

Step-5: BMD and Net BMD





Note:- Remaining portion are +ve and cutting portion are taken as -ve

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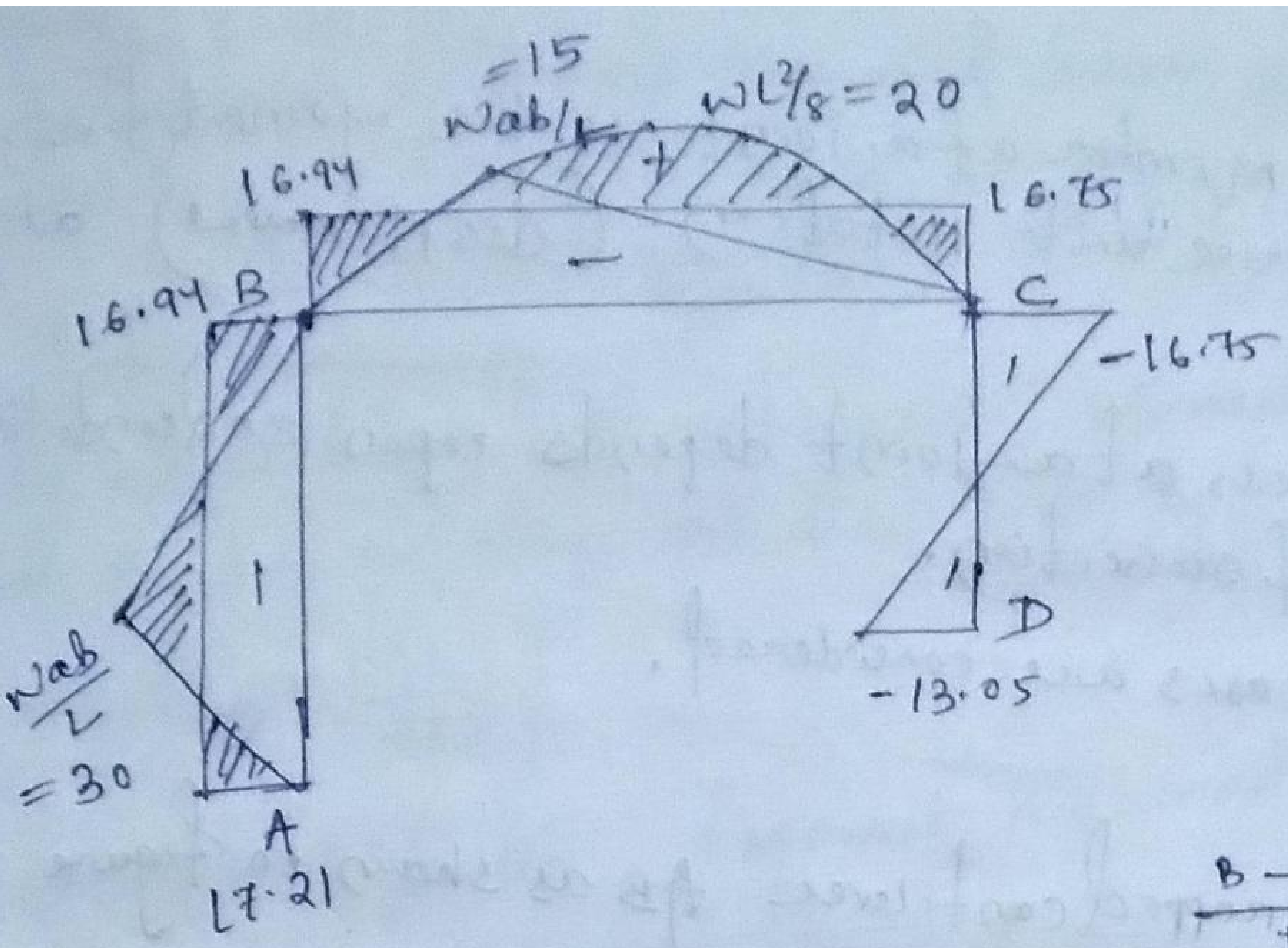
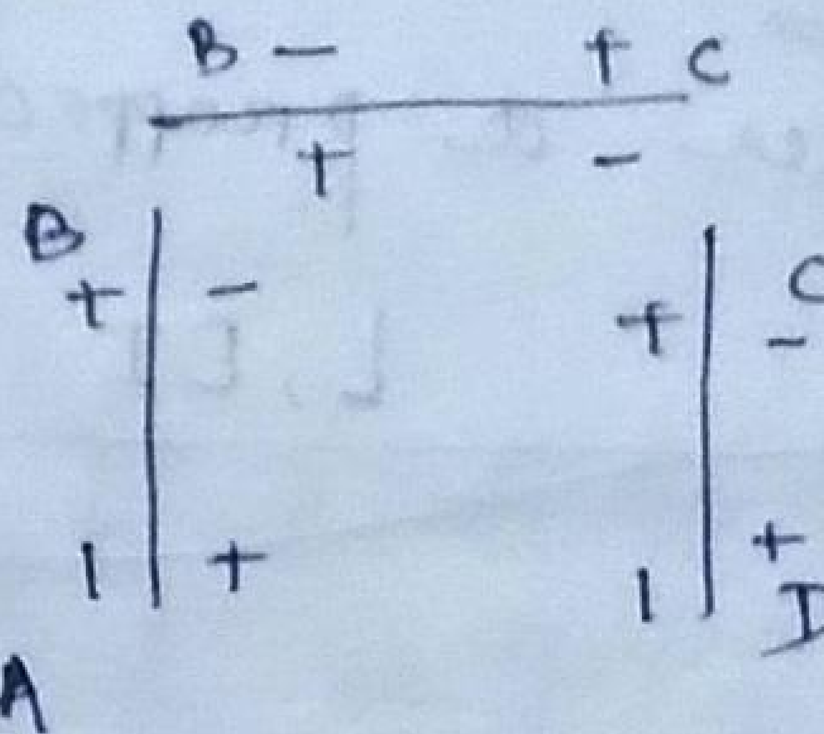




Note:- Remaining portion are +ve.
and cutting portion are taken as -ve.

Chapter-2

Moment Distribution Method

It is a displacement method of analysis of kinematically indeterminate structure.
It is based on stiffness approach.

→ It is based on stiffness approach.

→ Moment distribution method is not exact method but a method of successive approximation where any degree of accuracy can be obtained by repeated iteration.

→ This method was suggested by Prof. Hardy cross in 1930.

→ It is also known as Hardy cross method.

Stiffness :-

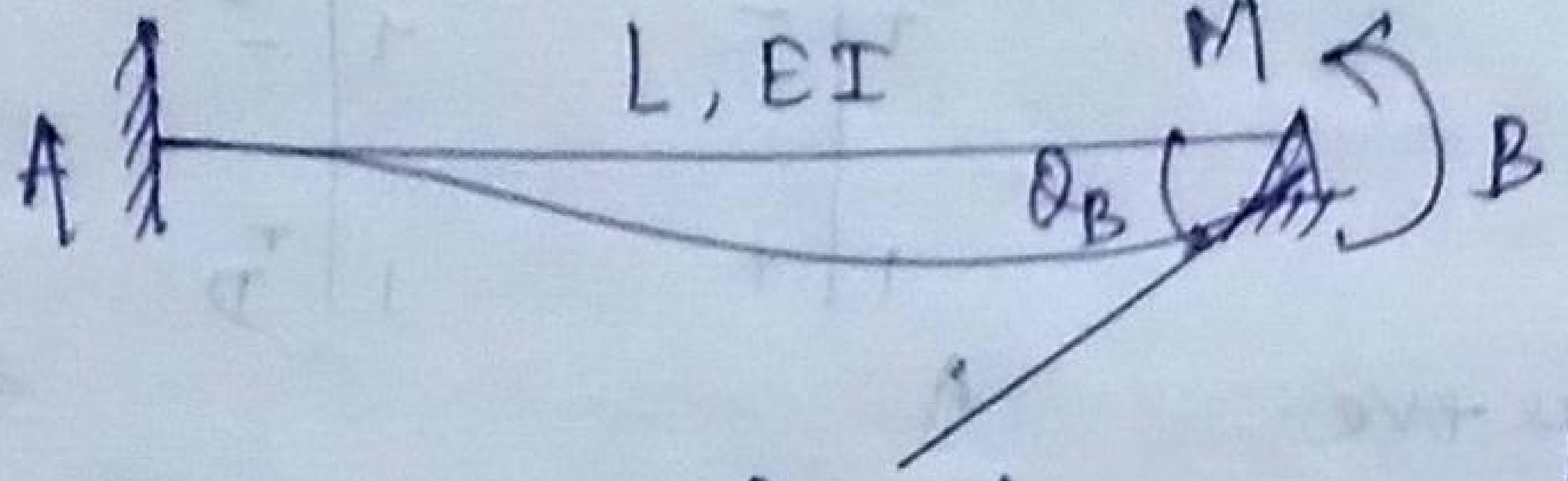
Stiffness for a member at a joint is the moment (force) required to produce unit rotation (displacement) at that joint.

Note :- Stiffness at a joint depends upon end conditions and properties of cross section.

for stiffness two cases are considered.

Case-1 :-

Consider a propped cantilever AB as shown in figure.



for above propped cantilever, if anticlockwise moment is applied at end B Then rotation at B is given by

$$\left[\theta_B = \frac{ML}{4EI}\right].$$

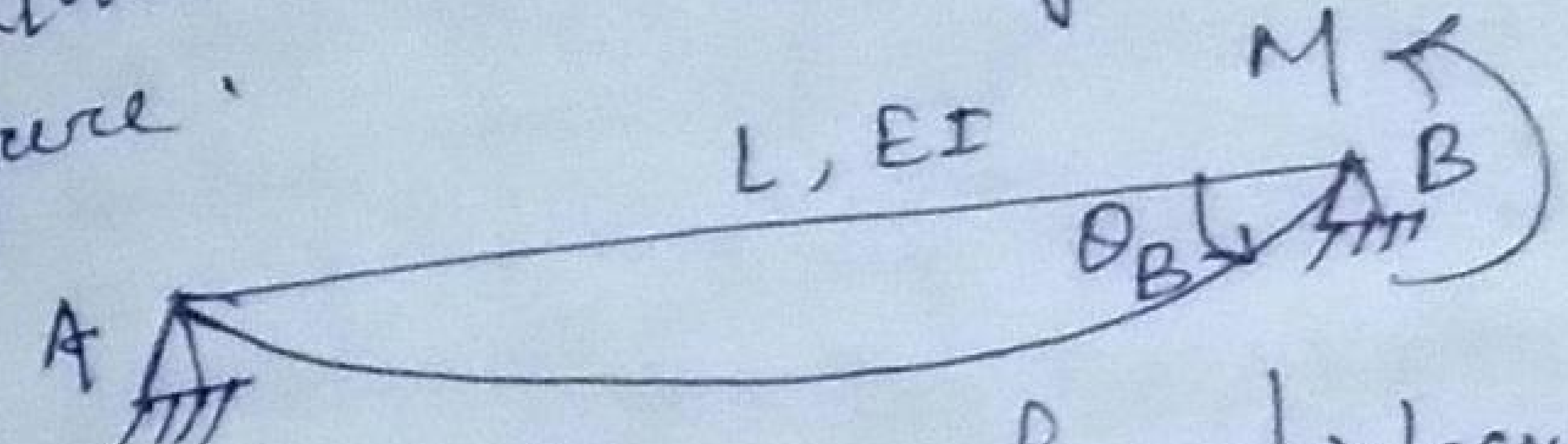
$$\Rightarrow \boxed{M = \frac{4EI}{L}\theta_B}$$ (Moment required to produce θ_B)

We know ($\theta_B = 1$) Then moment required to produce unit rotation is known as stiffness.

$$\boxed{K_{BA} = 4EI/L} \quad \because \left[M/\theta_B = K\right].$$

Case-2 :-

Now consider a simply supported beam AB as shown in figure.



for the above beam if anticlockwise Moment M is applied at end B, then rotation at B is given by.

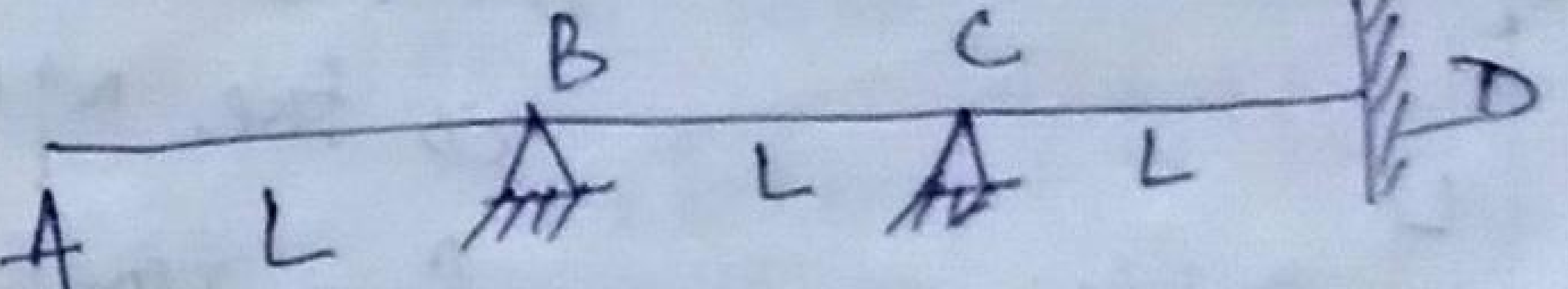
$$\boxed{\theta_B = \frac{ML}{3EI}}$$

$$\Rightarrow M = \frac{3EI}{L}\,\theta_B$$ [Moment required to produce θ_B]

if ($\theta_B = 1$) then $\boxed{K_{BA} = 3EI/L}$

Note:- Therefore it can be say that stiffness of a member when farther end is fixed is given by $4EI/L$ and stiffness of a member when farther end is hinged is given by $3EI/L$. and stiffness of a member when farther end is free is $\boxed{\text{zero}}$.

Example

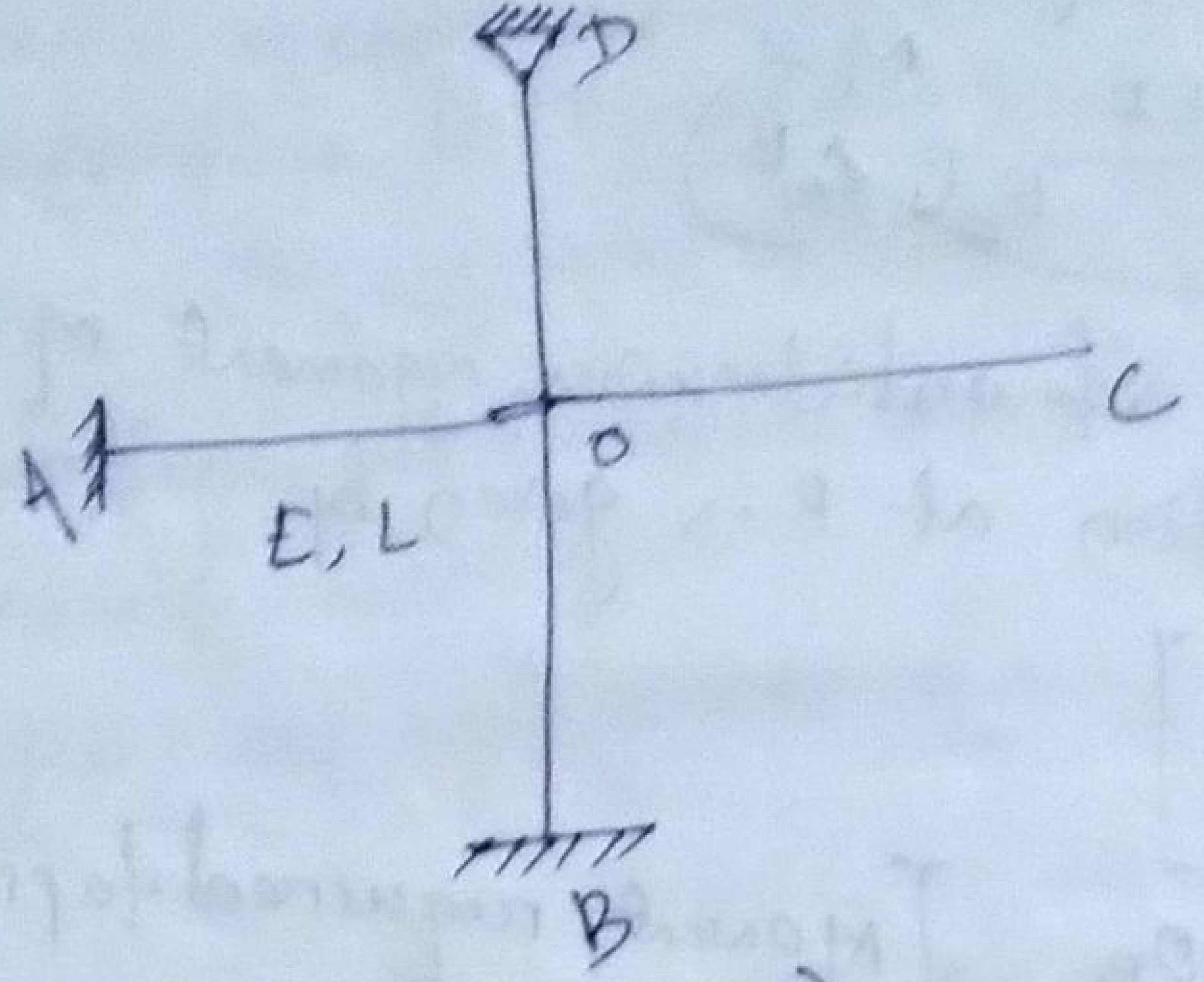


$K_{BA} = 0$ (farther end is free).

$K_{CB} = 3EI/L$ (farther end B is hinged).

$K_{CD} = 4EI/L$ (farther end D is fixed).

Consider a rigid jointed from figure shown below



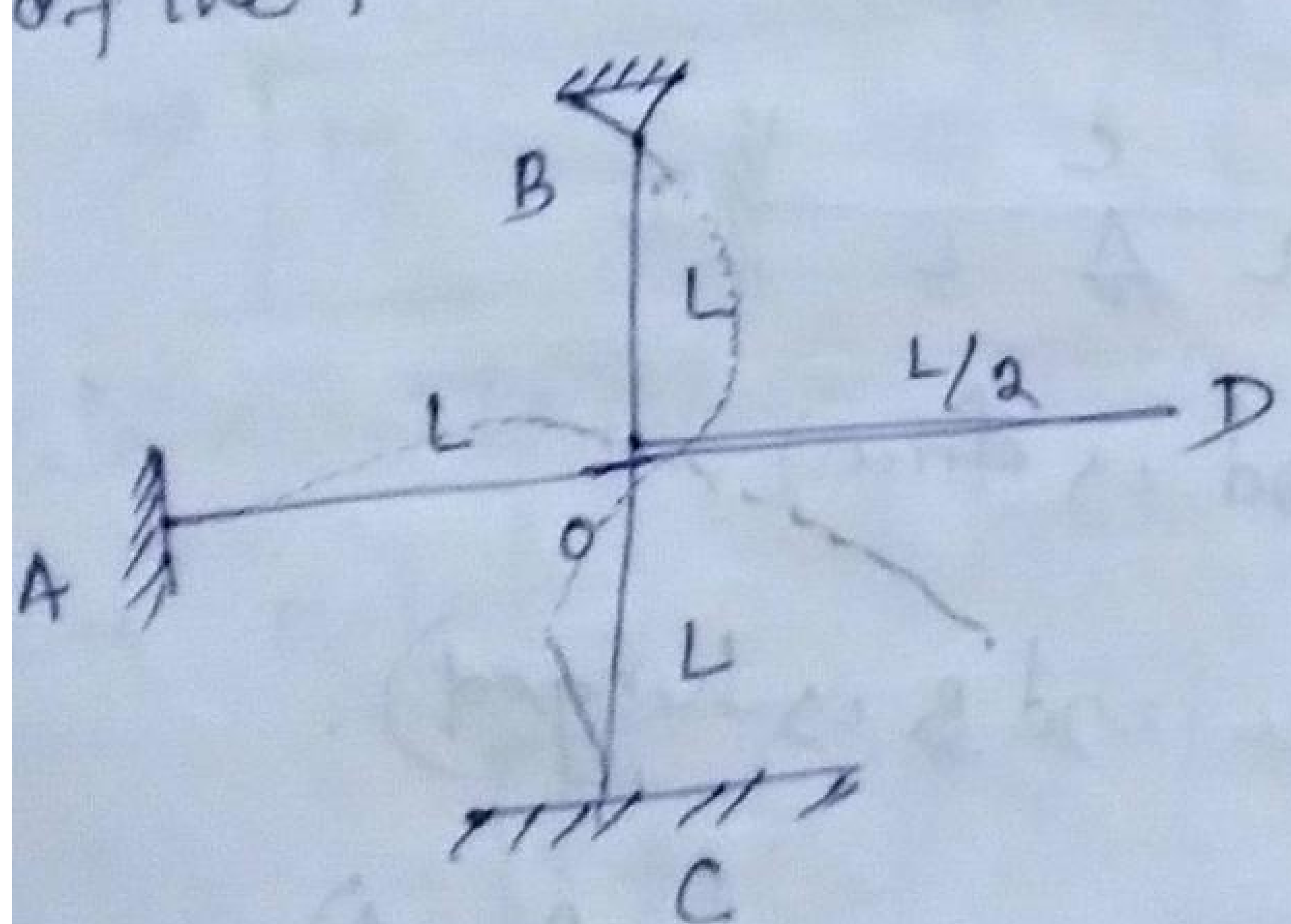
$K_{OA} = 4EI/L$ (as farend fixed)

$K_{OB} = 4EI/L$ (farend fixed)

$K_{OC} = 0$

$K_{OD} = 3EI/L$ (as farend hinged)

Q. A steel frame is shown in the given figure if joint 'O' of the frame is rigid, the rotational stiffness of the frame is ______



The rotational stiffness is the moment required for unit rotation at O

$$K_O = K_{OA} + K_{OB} + K_{OC} + K_{OD}$$

$$= \frac{4EI}{L} + \frac{3EI}{L} + \frac{4EI}{L} + 0$$

$$\boxed{K_O = 11EI/L}$$

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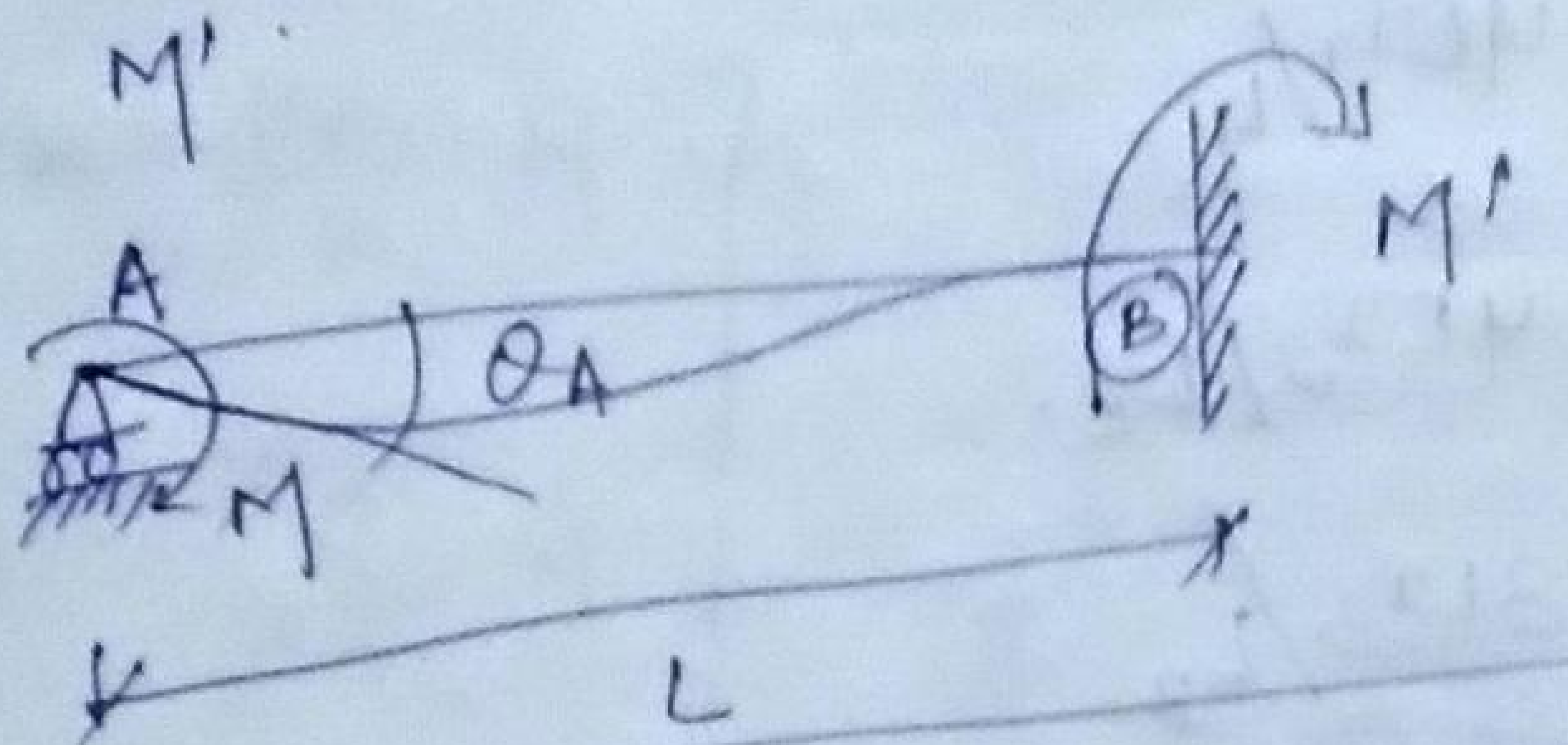
Carry over factor :-

It is defined as the ratio of the moment at the fixed far end to the moment at the rotating near end.

→ It is something which is being carried from one end to other end. by the member

Consider a beam AB of span L shown in figure. In this beam moment M is applied at end A where rotation is permitted while end B is fixed.

→ For applied moment M at A the moment developed at B is M'



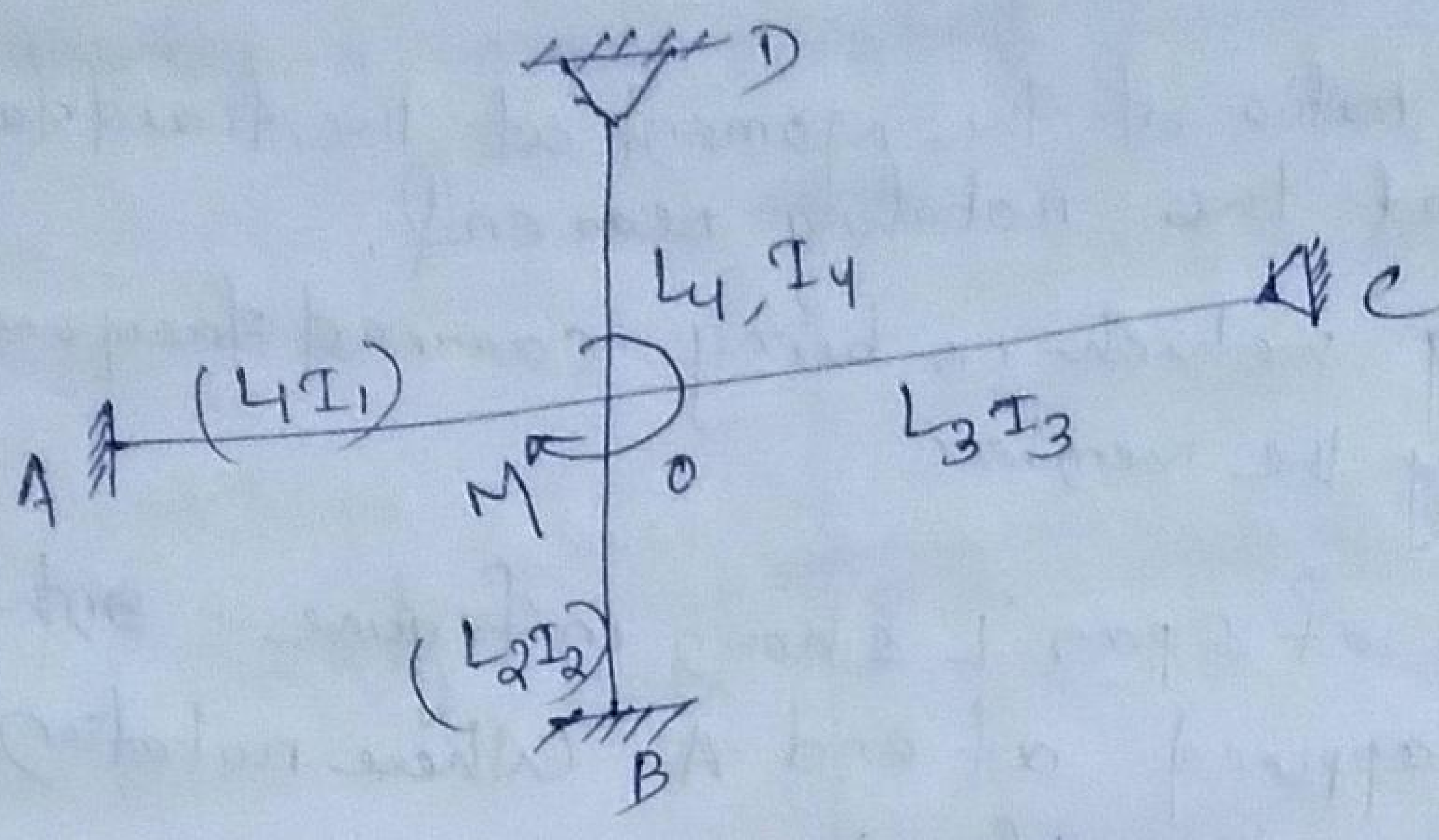
$\left[\text{Carry over factor} = M'/M\right]$

$$\boxed{\text{Carry over factor} = \frac{\text{Carry over moment at far end}}{\text{Applied moment at near end}} \quad \left(COF = M'/M\right)}$$

Expression for Distribution factor

Distribution factor for a member at a joint is the ratio of the stiffness (relative stiffness) of the member to the total stiffness (Total relative stiffness) of all the members ~~mette~~ meeting at that joint

$$\boxed{\text{Distribution factor (DF)} = \frac{\text{Stiffness of the member}}{\text{Total stiffness of the joint}}}$$



Consider the rigid jointed frame as shown in figure. E = constant and joint 'O' is rigid.

Stiffness of OA = $K_1 = 4EI_1/L$

Stiffness of OB = $K_2 = 4EI_2/L_2$.

Stiffness of OC = $K_3 = 3EI_3/L_3$

Stiffness of OD = $K_4 = 3EI_4/L_4$.

Total stiffness of the joint = $\Sigma K = K_1 + K_2 + K_3 + K_4$

$$= \frac{4EI_1}{L} + \frac{4EI_2}{L_2} + \frac{3EI_3}{L_3} + \frac{3EI_4}{L_4}.$$

DF for member OA at joint O = $\frac{\text{Stiffness of OA at joint (O)}}{\text{Total stiffness at joint}}$

$$(D.F)\ OA = \frac{K_1}{\Sigma K}.$$

$$(D.F)\ OA \text{ at joint}(O) = \frac{K_2}{\Sigma K}$$

D.F of member OB at joint O = $\frac{K_3}{\Sigma K}$.

D.F of member OB at joint O = $\frac{K_4}{\Sigma K}$.

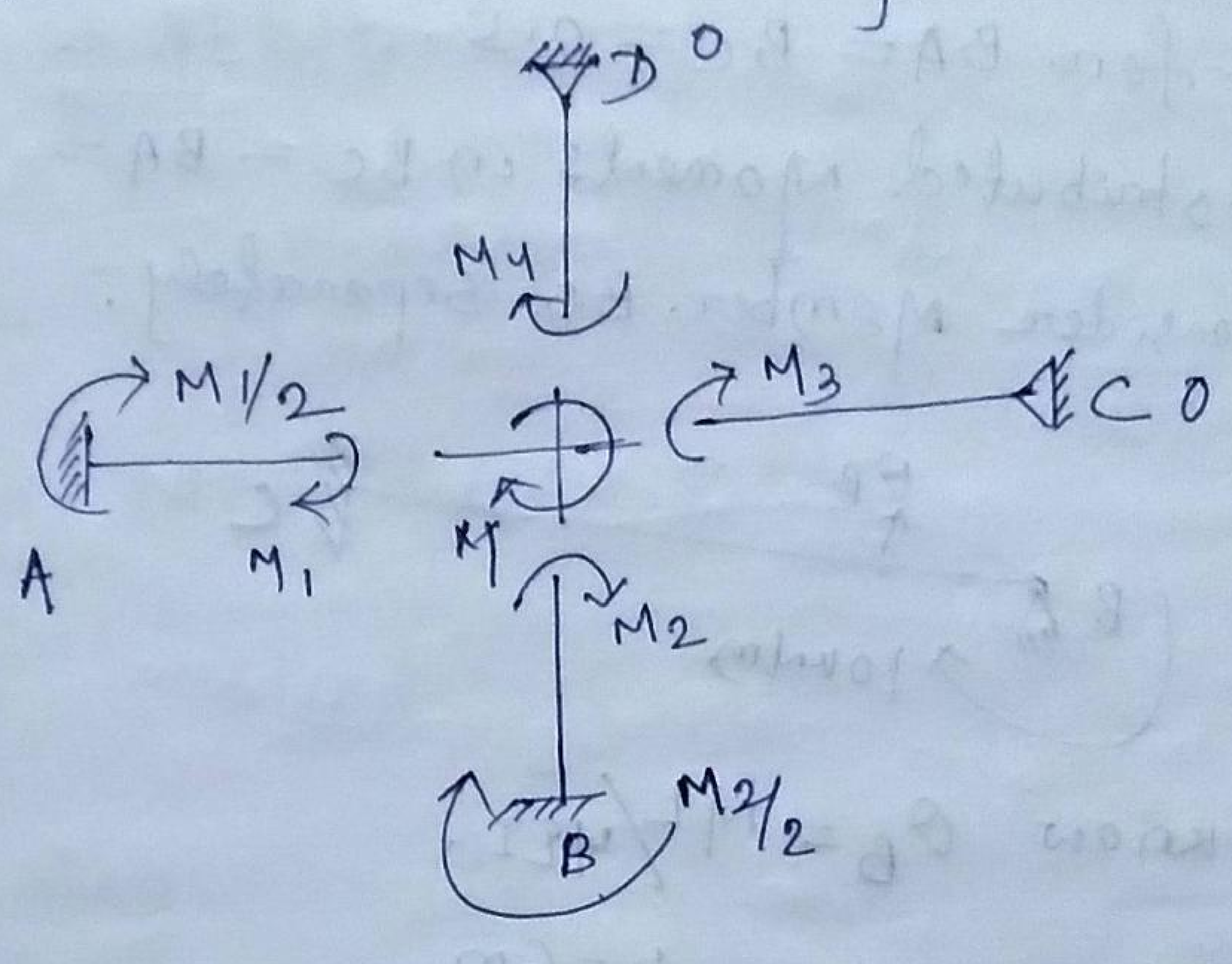
Note.

$$\boxed{M = M_1 + M_2 + M_3 + M_4}$$

Applied Moment 'M' is distributed in all members in the proportions of their stiffness.

let M_1, M_2, M_3 and M_4 are the moment distribution in the members OA, OB, OC, OD respectively at joint O.

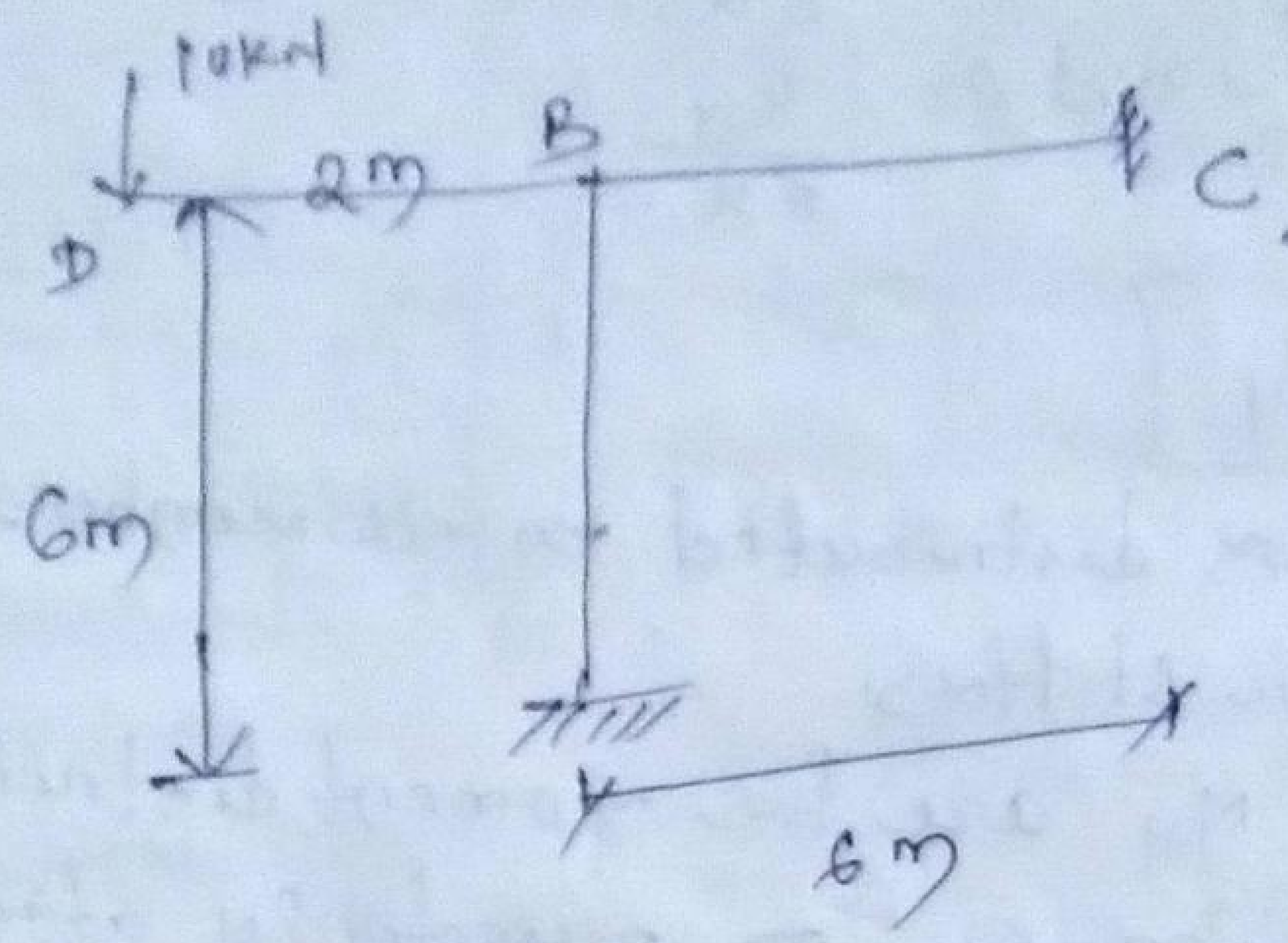
$$\left[\begin{array}{l} M_1 = \frac{K_1}{\Sigma K} M \\ \\ M_2 = \frac{K_2}{\Sigma K} M \\ \\ M_3 = \frac{K_3}{\Sigma K} M \\ \\ M_4 = \frac{K_4}{\Sigma K} M \end{array}\right]$$



Note: Distribution factors are the property of ~~rigid joint~~ hinge joint. So distribution factor of a hinge joint is always zero.

Q. Find the value of θ_B for the beam shown in figure below.

find the value of θ_B.

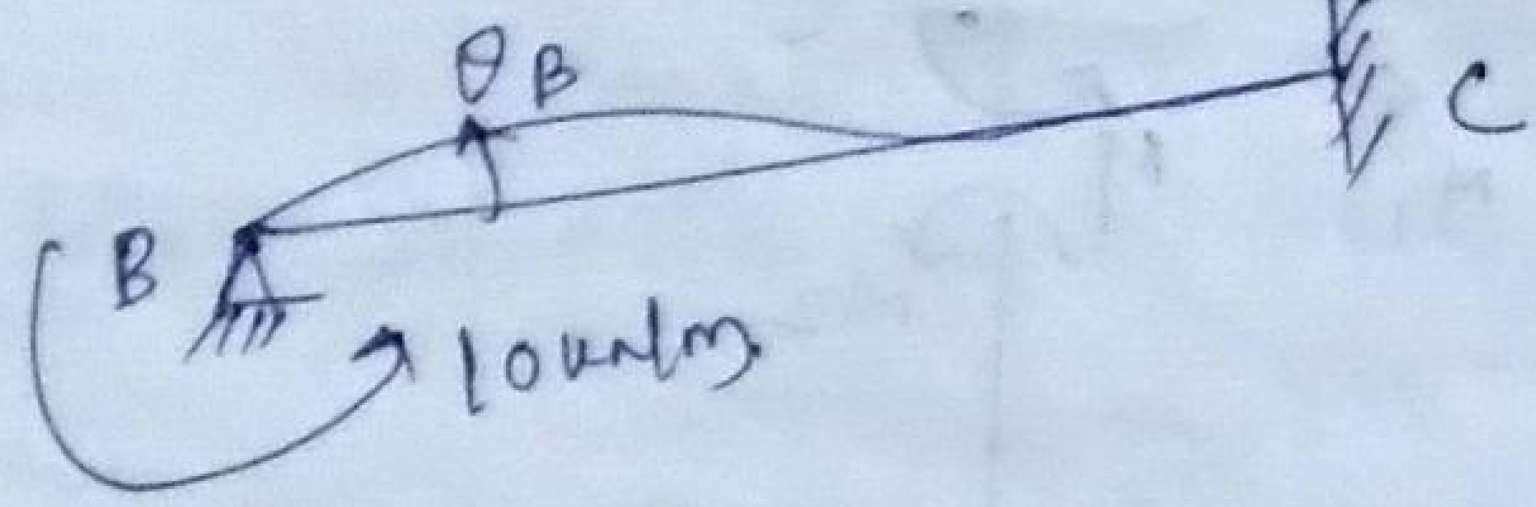


$\left[\begin{array}{l}\text{Moment} = 10\times2 \\ = 20m\end{array}\right]$

D.f for BA = BC = 0.5.

Distributed Moments in BC = BA = $0.5\times20 = 10$ KNm.

Consider Member BC separately.

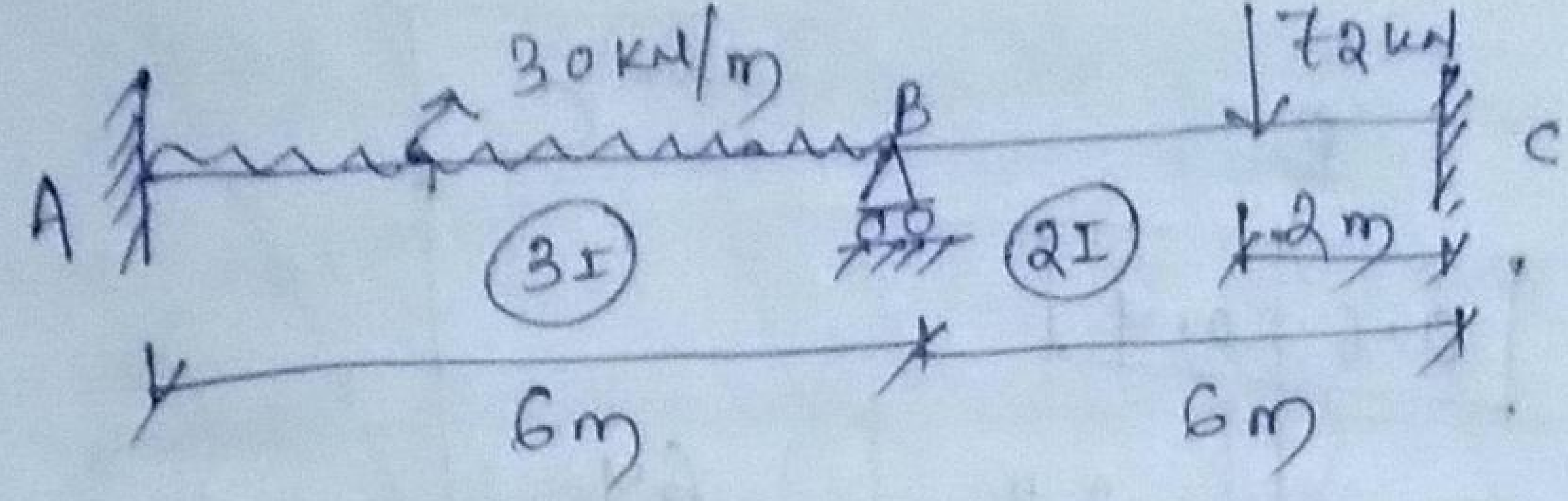


We know $\theta_B = ML/4EI$.

$M = 10$ KNm, $L = 6m$

$$\boxed{\theta_B = \frac{ML}{4EI} = \frac{10\times6}{4EI} = \frac{15}{EI}}$$

Q Analyse the continuous beam as shown in figure by Moment distribution method and draw Bending Moments and Shearforce diagrams. Draw elastic curve also.



Solution. Step-1:

Fixed end moment.

$$M_{FAB} = -wL^2/12 = \frac{-30\times 6^2}{12} = -90\,kNm.$$

$$M_{FBA} = +wL^2/12 = +90\,kNm$$

$$M_{FBC} = -\frac{wab^2}{L^2} = -72\times 4\times 2^2/6^2 = -32\,kNm$$

$$M_{FCB} = +wa^2b/L^2 = 72\times 4^2\times 2/6^2 = 64\,kNm.$$

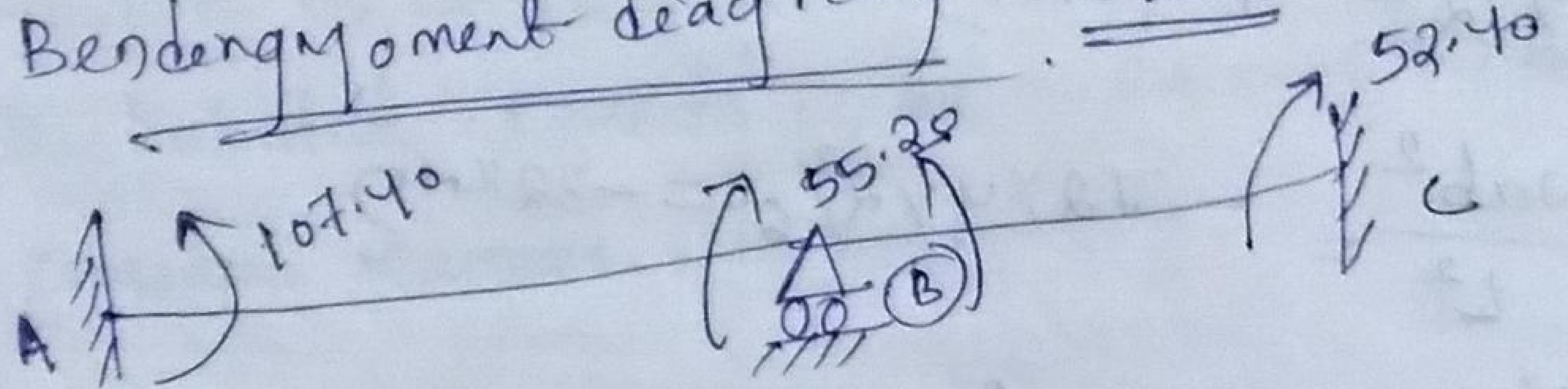
Distribution factors. Step-2

Joint	Members	K	ΣK	D.F
B	BA	$\frac{4EI}{L} = 2EI$	3.33EI	0.6
	BC	$\frac{4E(2I)}{6} = 1.33EI$		0.4

Step-3 Moment distribution Table

A	B		C
	0.6	0.4	
-90	90	-32	64
	-34.8	-23.2	
-17.40			-11.60
-107.40	55.20	-55.20	52.40

Bendingmoment diagram. **Step-3**

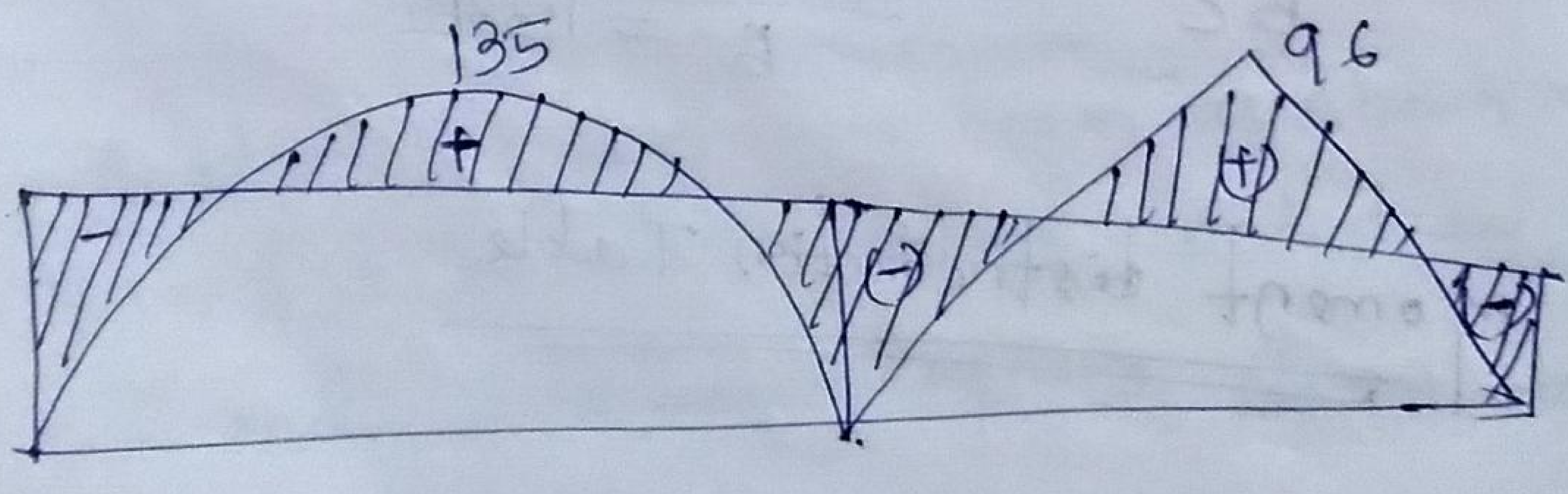


for AB beam

$$wl^2/8 = 30 \times 6^2/8 = 135 \text{ kN/m}.$$

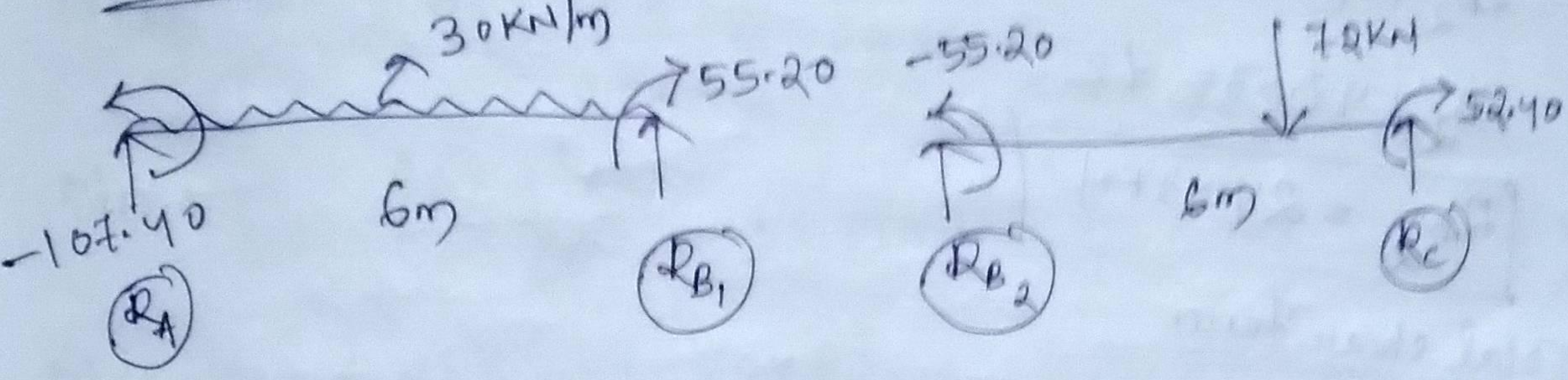
for BC beam

$$Wab/L = \frac{72 \times 4 \times 2}{6} = 96 \text{ kNm}.$$



(BMD)

Step:- 4



$\Sigma M_A = 0$

$\Rightarrow -R_B \times 6 - 107.40 + 55.20 + 30 \times 6 \times 6/2 = 0$

$\Rightarrow -R_B \times 6 = 487.8\,KN$

$\boxed{R_{B_1} = 81.3\,KN}$

upward force = downward force

$\Rightarrow R_A + R_B = 30 \times 6$

$\Rightarrow R_A + 81.3 = 180$

$\Rightarrow \boxed{R_A = 98.7\,KN}$

for Beam BC :

$\Sigma M_B = 0$

$\Rightarrow -R_C \times 6 - 55.20 + 52.40 + 72 \times 4 = 0$

$\Rightarrow -R_C \times 6 = 285.2$

$\Rightarrow \boxed{R_C = 47.53\,KN}$

upward force = downward force

$\Rightarrow RB_2 + R_C = 72$

$\Rightarrow RB_2 + 47.53 = 72$

$\Rightarrow \boxed{RB_2 = 24.47\,KN}$

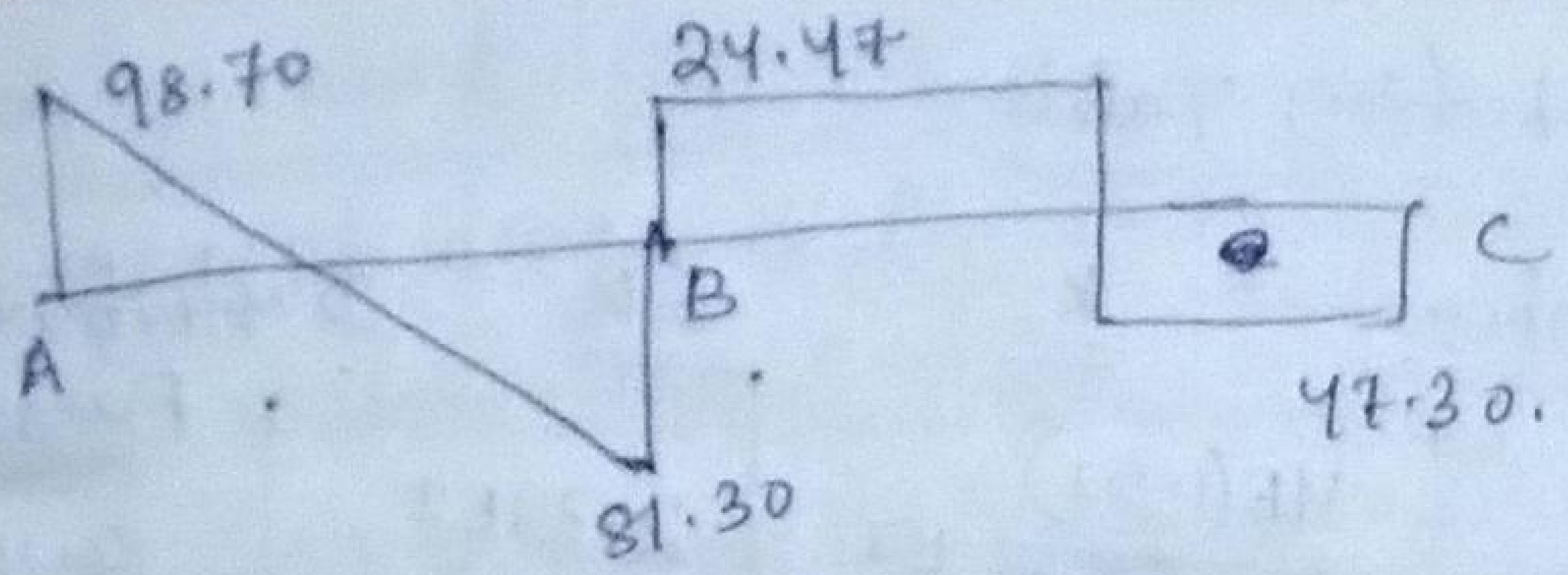
Total shear force

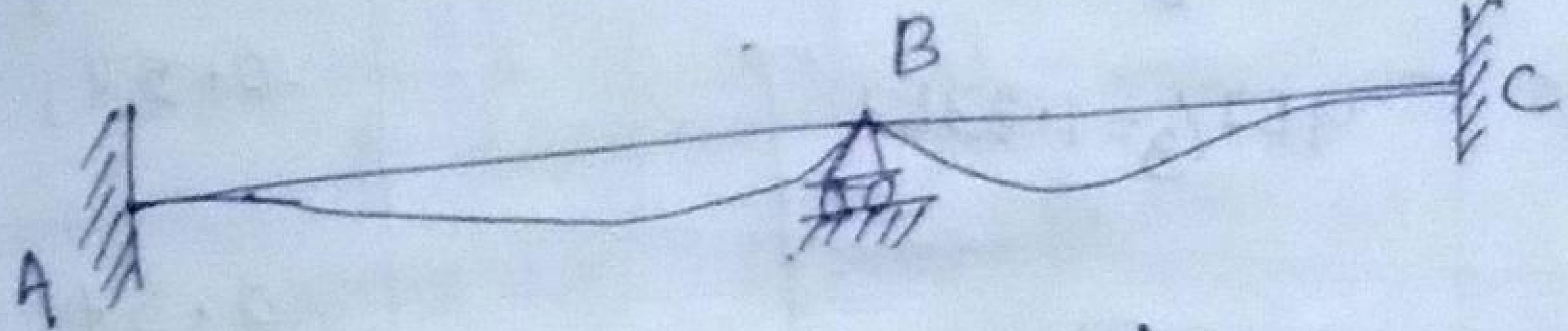
$R_A = 98.7\,KN$

$R_B = RB_1 + RB_2 = 24.47 + 81.3 = 106\,KN.$

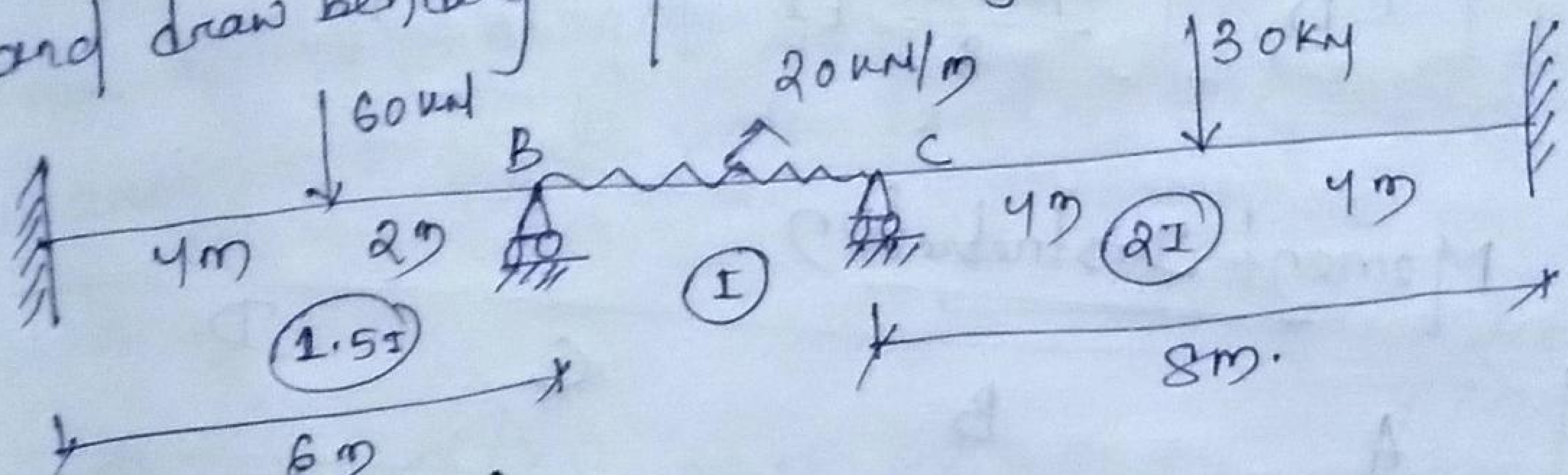
$R_C = 47.53\,KN$

Hence shear force diagram will be as shown in figure





Q Analyse the continuous beam as shown in figure and draw bending moment diagram.



Solution:- fixed end Moment.

$$M_{FAB} = -\frac{60\times4\times2^2}{6^2} = -26.67\ \text{kNm.}$$

$$M_{FBA} = \frac{60\times4^2\times2}{6^2} = 53.33\ \text{kNm.}$$

$$M_{FBC} = -wl^2/12 = -\frac{20\times3^2}{12} = -15\ \text{kNm}$$

$$M_{FCB} = +wl^2/12 = +15\ \text{kNm}$$

$$M_{FCD} = -wl/8 = \frac{-30\times8}{8} = -30\ \text{kNm,}$$

$$M_{FDC} = +30\ \text{kNm.}$$

Step-2:- Distribution Table

Joints	Members	K	ΣK	Distribution Factor
B	BA	$\frac{4E(1.5I)}{6} = EI$	2.33EI	0.429
	BC	$4EI/3 = 1.33EI$		0.571
C	CB	1.33 EI	2.33 EI	0.571
	CD	$\frac{4E(2I)}{8} = EI$		0.429

Step: 3 Moment Distribution

Joints	A	B		C		D
		0.429	0.571	0.571	0.429	
		53.33	-15	15	-30	30
	-26.67	-16.44	-21.89	8.57	6.43	
	-8.22	0	4.29	-10.95	0	3.22
		-1.84	-2.45	6.25	4.70	2.35
	-0.92	0	3.13	-1.23		
		-1.34	-1.79	0.70	0.53	0.27
	-0.67	0	0.35	-0.19		
		-0.15	-0.20	0.51	0.39	0.20
	-0.8		0.26	-0.1		
		-0.11	-0.115	0.06	0.04	
Final Moment	36.56	33.45	-33.45	17.91	-17.91	36.04

Scanned by TapScanner

Bending Moment diagram

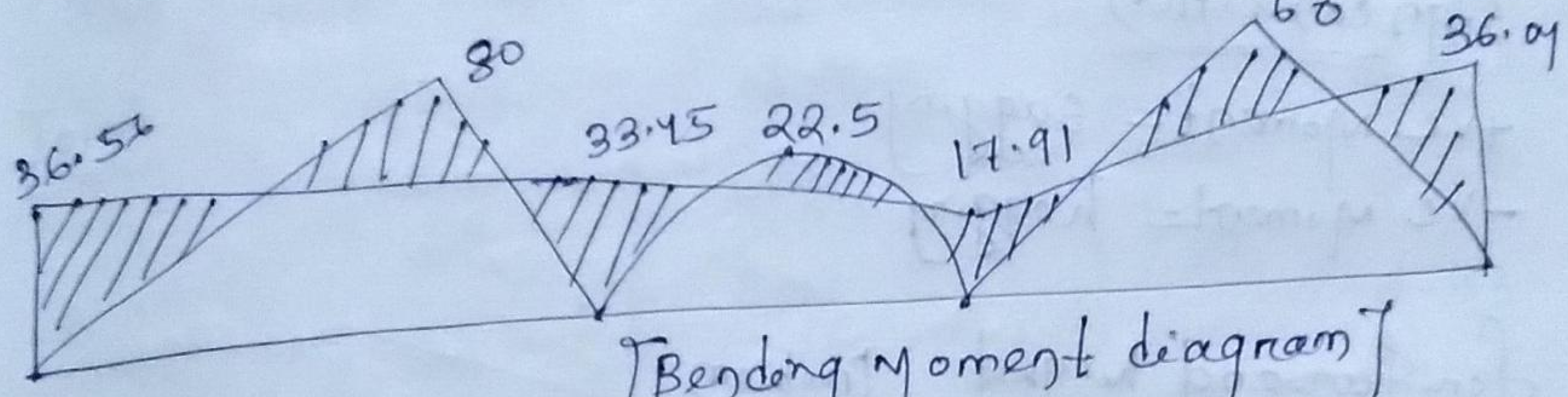
Free Moment diagram for AB is a triangle with Maxm ordinate under the Load.

$$= \frac{Wab}{l} = \frac{60 \times 4 \times 2}{6} = 80 \text{ KNm}$$

Free Moment diagram for BC is a symmetric parabola with Maximum ordinate

$$= wl^2/8 = 20 \times 3^2/8 = 22.5 \text{ KNm}.$$

Free Moment diagram for CD is a triangle with Maximum ordinate under Load $= Wl/4 = \frac{30 \times 8}{4} = 60 \text{ KNm}$



[Bending Moment diagram]

* Procedure of Analysis by Moment distribution method

Step-1:- Find fixed end Moment for each member considering each end to be fixed.

Sign convention for fixed end Moment

clockwise Moment = +ve.

Anticlockwise Moment = −ve.

Step-2:- Find distribution factors for all members meeting at a joint. Each joint is considered rigid.

Step-3:- Find unbalance Moment at each joint.
Find the balancing Moment at each joint.
Distribute the balancing Moment according to their distribution factor and transfer carry over Moments to their farther end of farther ends are fixed.

Step-4:- Find final end Moments at the ends when all joint are balanced.

Step-5:- Draw BMD for given loading by stiffness approach as discussed.

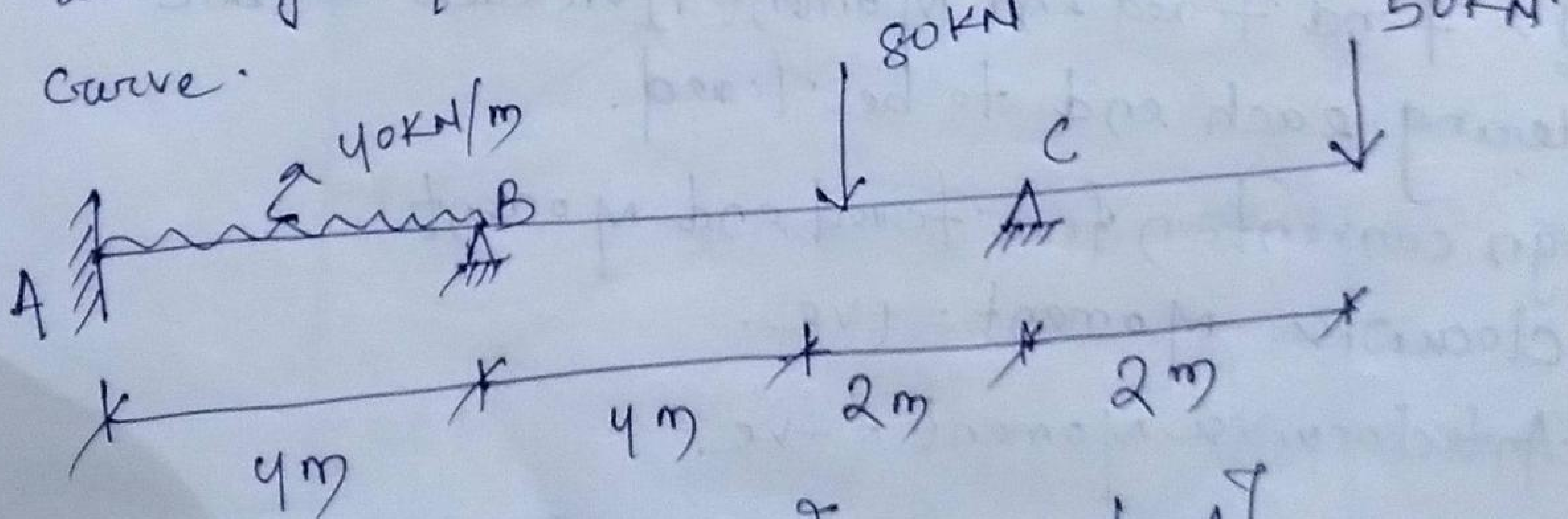
Sign convention

+ve Moment = sagging

-ve Moment = hogging

For far end hinged **Problem**

Q Analyse the continuous beam as shown in figure below by Moment distribution method. and elastic curve.



[EI is constant]

Solution:- Fixed end moment

$$M_{FAB} = -wL^2/12 = \frac{-40\times4^2}{12} = -53.33 KNm$$

$$M_{FBA} = +wL^2/12 = \frac{+40\times4^2}{12} = +53.33 KNm.$$

$$M_{FBC} = -wab^2/L^2 = \frac{-80\times4\times2^2}{6^2} = -35.56 KNm$$

$$M_{FCB} = \frac{+wa^2b}{L^2} = \frac{+80\times4^2\times2}{6^2} = +71.11 KNm.$$

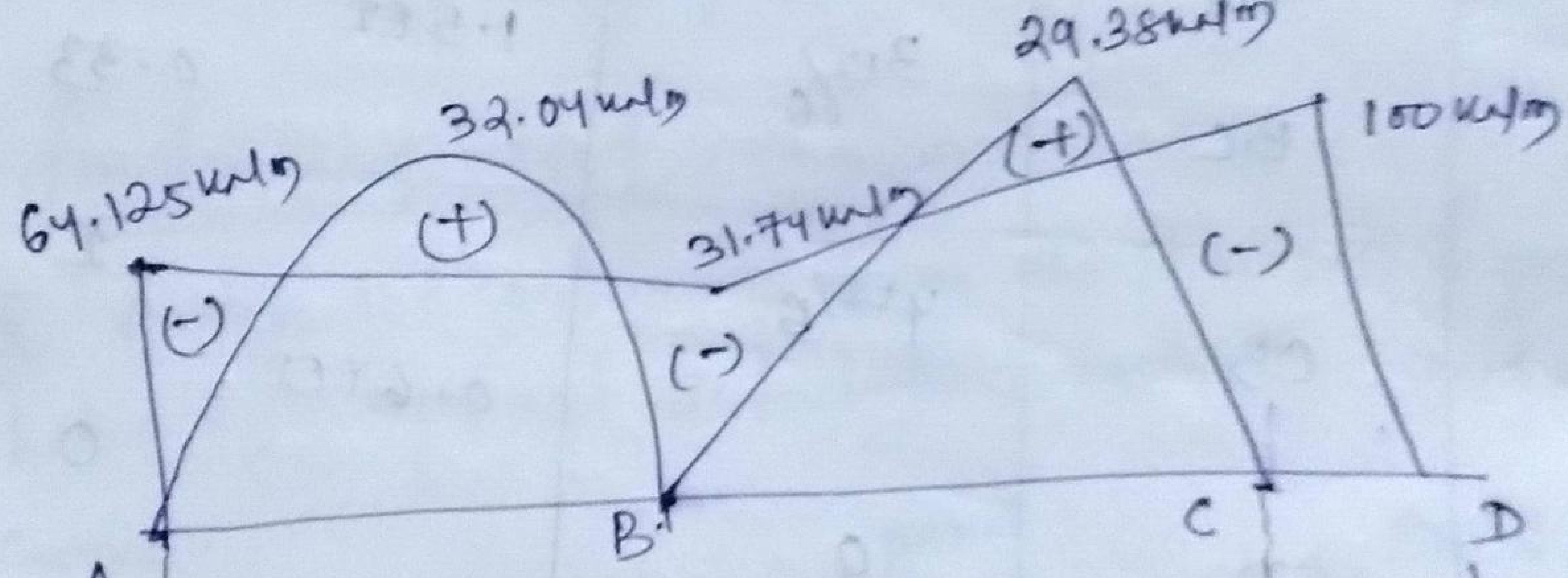
$$M_{FCD} = -50\times2 = -100 KNm.$$

Joint	Member	Stiffness	Total stiffness	D.F
B	BA	4EI/4	1.5 EI	0.67
	BC	3EI/6		0.33
C	CB	4EI/6	0.67EI	1
	CD	0		0

Step:- 3

Moment Distribution Table

	A	B		C		D
		1	0	1	0	
fem	-53.33	+53.33	-35.56	+71.11	-100	0
Balancing Moment		-11.90	-5.87	+28.89		
COM	-5.96	0	14.45			
Balancing Moment		-9.69	-4.76			
COM	-4.845					
Final endMoment	– 64.125	31.74	-31.74	100	-100	0



[Elastic curve]